Roy, A. and Chakraborty, S. (2024) Seismic reliability analysis of structures by an adaptive support vector regression based metamodel. *Journal of Earthquake Engineering*, 28(6), pp. 1590-1614. (doi: 10.1080/13632469.2023.2242975)

This is the author accepted version of the article.
There may be differences between this version and the published version.
You are advised to consult the published version if you wish to cite from it:
https://doi.org/10.1080/13632469.2023.2242975

https://eprints.gla.ac.uk/303640/

Deposited on: 31 July 2023

Enlighten – Research publications by members of the University of Glasgow
http://eprints.gla.ac.uk

Seismic Reliability Analysis of Structures by an Adaptive Support Vector Regression-based Metamodel

Atin Roy[a,b] and Subrata Chakraborty[a]*

[a]*Department of Civil Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah, India.*

[b]*James Watt School of Engineering (Infrastructure and Environment), University of Glasgow, Glasgow, UK.*

Department of Civil Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India. e-mail: schak@civil.iiests.ac.in *corresponding author.

Seismic Reliability Analysis of Structures by an Adaptive Support Vector Regression-based Metamodel

The dual metamodeling approach is usually adopted to tackle the stochastic nature of earthquakes in seismic reliability analysis relying on the lognormal response assumption. Alternatively, a direct response approximation approach where separate metamodels are constructed for each earthquake is attempted here avoiding prior distribution assumption. Further, an adaptive support vector regression-based metamodeling is proposed that selects new training samples near the failure boundary with due consideration to accuracy and efficiency. The effectiveness of the approach is elucidated by comparing it with the results obtained by the direct Monte Carlo simulation technique and a state-of-the-art active learning-based Kriging approach.

Keywords: Seismic Reliability Analysis, Metamodel, Support Vector Regression, Adaptive Sampling, Monte Carlo Simulation.

1. Introduction

The performance-based earthquake engineering (PBEE) is receiving much importance nowadays for seismic safety assessment of structures due to its ability to integrate the effect of the stochastic nature of earthquakes and uncertainty about various system parameters characterizing the behaviour of a structure (Porter, Kennedy, and Bachman 2007). Basically, the approach conducts seismic reliability analysis (SeRA) of a structure, i.e., obtains the probability that the seismic demand of the structure exceeds its capacity for a target hazard level. From the structural engineering viewpoint, SeRA is primarily a time-varying reliability analysis problem for a given limit state function (LSF). Thus, SeRA mainly requires solving an outcrossing problem in classical random vibrations theory considering seismic motion as a stochastic process. However, statistical variations of seismic responses obtained by the random vibration approach are noted to be significantly lower than the response variances obtained by the more accurate direct Monte Carlo simulation (MCS) technique (Pinto 2001). Furthermore, the frequency domain approach typically applied in the random vibration setup may not be appropriate for SeRA in the PBEE framework as the approach involves the nonlinear response behaviour of a structure. The reliability analysis problem in the PBEE framework is usually simplified to obtain the failure probability that the maximum seismic response exceeds an allowable value over the entire duration of an earthquake (Buratti, Ferracuti, and Savoia 2010) to ensure a specific performance level. Hence, the LSF is expressed as,

$$g(\mathbf{X}) = \min_t \left[C(\mathbf{X}_C, t) - D(\mathbf{X}_D, t) \right] \tag{1}$$

In the above, LSF is based on the difference between the seismic demand (D) and capacity (C) of a structure considering uncertainty due to the vectors $\mathbf{X}_C$ and $\mathbf{X}_D$ representing the seismic demand and capacity-related random variables, respectively and t is the time parameter. The primary task of SeRA is to estimate the failure probability based on an LSF as described by Eq. (1). The failure occurs when $g(\mathbf{X}) < 0$ and the probability of failure can be obtained based on the available reliability analysis methods. The analytical method is most widely adopted (Günay and Mosalam 2013) for its efficiency. However, the FORM-based analytical approach requires an assumption of the shape of a performance function. Moreover, its level of accuracy may not be acceptable, particularly when the magnitude of randomness is large. The brute-force MCS technique is noted to be quite simple in concept and the most accurate in this regard. This approach is most preferred for the SeRA of structures without the assumption of the specific distribution of involved LSFs. However, the approach needs to perform a large number of repetitive nonlinear dynamic seismic response analyses (NDSRA) to obtain an acceptable number of sample responses (Kwon and Elnashai 2006) for meaningful statistical analysis. The metamodeling approach has emerged as a viable alternative to alleviate such computational burden while conserving the maximum possible accuracy.

The application of the metamodeling approach is quite beneficial in replacing the complex model of a structure to largely reduce the number of NDSRAs involved in the brute force MCS technique. The application of the MCS technique in the framework of metamodeling-based reliability analysis methods is well known (Bucher and Bourgund 1990). The use of the polynomial response surface method (RSM) for structural reliability analysis (SRA) is the simplest approach (Bucher and Bourgund 1990). Various advanced metamodeling techniques like artificial neural networks (ANN), Kriging, support vector machine (SVM), multivariate adaptive regression splines (MARS), radial basis function networks (RBFN) etc. are frequently employed for SRA efficiently (Afshari et al. 2022; Xu and Saleh 2021). Among different metamodeling approaches, the SVM-based metamodels are founded on the structural risk minimization principle and small sample learning theory. As a result, these metamodels have the generalization capability in approximating an implicit function (Vapnik 1998; 2000). In the early studies, the SVM-based classification approach was utilized for SRA (Hurtado 2004; 2007; Rocco and Moreno 2002). The application of support vector regression (SVR), an extension of SVM for regression that can overcome the curse of dimensionality, is quite notable for SRA (H. S. Li et al. 2006; Moura et al. 2011; Dai et al. 2012; Richard, Cremona, and Adelaide 2012; Dai, Zhang, and Wang 2015; Bourinet 2016; Roy, Manna, and Chakraborty 2019; Roy and Chakraborty 2020; Keshtegar et al. 2021; Roy and Chakraborty 2022; 2023).

Regarding the application of the metamodeling approach, it is not an easy task for SeRA like metamodeling-based reliability analysis of structures under static and deterministic dynamic loads as discussed above where the random loads are represented by single or multiple but finite numbers of random variables. However, for the SeRA of structures, the stochastic nature of loading cannot be described by a finite number of parameters (i.e., random variables) to consider the record-to-record variation of earthquake ground motion. To evade this difficulty, the input variable can be separated into two groups i.e. the structural system parameters and the stochastic sequences e.g. SeRA by RSM with random factor (Buratti, Ferracuti, and Savoia 2010). Following such a concept, the dual RSM

(Lin and Tu 1995) was used for SeRA of structures (Towashiraporn 2004; Seo et al. 2012; Seo and Linzell 2013; Park and Towashiraporn 2014; Saha, Matsagar, and Chakraborty 2016; Gaxiola-Camacho et al. 2017; Shyamal Ghosh and Chakraborty 2017; Shyamal Ghosh, Ghosh, and Chakraborty 2018; Zhang and Wu 2019; Yan Xiao, Ye, and He 2020). However, such dual metamodeling approach for SeRA of structures requires prior distributions assumption of seismic responses. Alternatively, metamodels are also constructed by decomposing the nonlinear input-output relation with high-dimensional model representation (HDMR) to approximate the relation between seismic responses and uncertain inputs of the structural models for SeRA (Unnikrishnan, Prasad, and Rao 2013; Zentner and Borgonovo 2014). On the other hand, earthquake accelerations are also used directly as inputs to the metamodel to predict structural response time histories (Mai et al. 2016). Recently, a similar approach has been explored for SeRA of structures using deep neural networks (Kundu, Ghosh, and Chakraborty 2022).

The application of metamodeling approaches in SeRA of structures is notable starting with the polynomial RSM (Franchin et al. 2003; Möller et al. 2009; Buratti, Ferracuti, and Savoia 2010; Seo et al. 2012; Seo and Linzell 2013; Park and Towashiraporn 2014; Saha, Matsagar, and Chakraborty 2016; Gaxiola-Camacho et al. 2017). The successful application of various advanced metamodeling techniques includes moving least-square method-based RSM (Shyamal Ghosh and Chakraborty 2017), ANN (Nikolaos D. Lagaros and Fragiadakis 2007; Nikos D. Lagaros et al. 2009; Wang et al. 2018), Kriging (Gidaris, Taflanidis, and Mavroeidis 2015; Shyamal Ghosh, Roy, and Chakraborty 2021), Bayesian networks (Gehl and D’Ayala 2016), Lasso regression (Mangalathu, Jeon, and DesRoches 2018), SVM (Sainct et al. 2020), SVR (Shyamal Ghosh, Roy, and Chakraborty 2018) etc. J. Ghosh, Padgett, and Dueñas-Osorio (2013) compared the performances of polynomial RSM, MARS, RBFN, and SVR for seismic vulnerability assessment of highway bridges. A comparative study of polynomial RSM, MARS, RBFN, SVR, adaptive basis function construction and random forest for regression is also worth noting for seismic fragility analysis of concrete gravity dams (Segura, Padgett, and Paultre 2020).

Most of the studies of metamodeling-based SeRA of structures employed a design of experiments (DOE) selected by one-shot sampling approaches where the sample size and training points from the input space of the considered random variables are determined once in a single stage. In this regard, the adaptive sampling approach has emerged to construct metamodel iteratively for improved SRA (Bucher and Bourgund 1990; Rajashekhar and Ellingwood 1993; Echard, Gayton, and Lemaire 2011; Richard, Cremona, and Adelaide 2012; Pan and Dias 2017; Bourinet 2016; Marelli and Sudret 2018; N. C. Xiao, Zuo, and Zhou 2018; X. Li et al. 2018; Cheng and Lu 2020; Roy and Chakraborty 2020; 2022; Ren et al. 2022). Teixeira, Nogal, and O'Connor (2021) surveyed the adaptive sampling strategies for metamodeling-based SRA. Among different adaptive sampling strategies, the branch of active learning-based SRA approach has drawn significant attention. An early use of the learning function for adaptive sampling is noted in efficient global optimization (Jones, Schonlau, and W. J. Welch 1998). The most applied active learning-based algorithms (Echard, Gayton, and Lemaire 2011; Bichon et al. 2008) and a recent review article on active learning in SRA (Moustapha, Marelli, and Sudret 2022) provide a complete overview of the field.

In this regard, it may be realized that an adaptive sampling scheme to construct a metamodel for SeRA is not an easy task. As already mentioned, the record-to-record variation of earthquake time history needs to be considered to replicate the stochastic nature of earthquake motion. It is important to note that each ground motion in the bin has separate failure surfaces for an LSF of interest at a particular intensity level. Thus, there could be variability in the locations of the failure planes due to such record-to-record variations of ground motions. Now, these failure surfaces shift when the intensity of the earthquake changes. Therefore, the reduced space to select the adaptive training samples is not unique for different intensity levels. Nevertheless, the reduced spaces are also different for different ground motions in the bin, as the responses corresponding to each ground motion are approximated by a separate metamodel. As already mentioned, there are many adaptive sampling strategies for metamodeling-based SRA, but most of the adaptive sampling schemes or active learning approaches of SRA deal with static reliability analysis of structure where all the uncertain parameters are modelled as scalar random variables. However, the input seismic load for metamodeling-based SeRA is no longer a scalar quantity but a vector process and stochastic. Hence, applying an active learning approach for SeRA is not straightforward like SRA analysis under static load as it involves a large number of input parameters due to the high-dimensional nature of earthquakes. There are attempts to apply active learning approaches for reliability analysis of structures under a deterministic seismic load (Zhou, Peng, and Li 2019; Roy, Chakraborty, and Adhikari 2023) which can be treated like traditional SRA problems. Thus, such studies are not considered in the context of metamodeling-based SeRA. To the best of our knowledge, a limited number of studies on adaptive sampling strategies for metamodeling-based SeRA are noted. Yanjie Xiao, Yue, and Zhang (2021) proposed an adaptive Gaussian process regression-based metamodeling approach for SeRA. Besides, there are few recent studies on active learning-based seismic fragility analysis e.g., an active learning-based dual metamodeling approach (Yanjie Xiao et al. 2022) and an active learning reliability approach using gradient boosting classifiers (Jeddi et al. 2022). However, these approaches involve prior distribution assumption of seismic responses like the usual dual metamodeling approach (Towashiraporn 2004). Thus, it seems to be important to develop an efficient adaptive sampling-based metamodeling approach for the SeRA of structures that avoids a prior distribution assumption of seismic response. In doing so, a direct metamodeling approach where responses of each ground motion are approximated by adaptive metamodels would be attempted. For adaptive metamodeling, adaptive SVR algorithms (Roy and Chakraborty 2020; 2022) could be employed. However, it can be realized that adopting such adaptive SVR algorithms (Roy and Chakraborty 2020; 2022) that are predominantly developed for reliability analysis of structures under static loads will involve a large number of actual function evaluations for SeRA where one needs to update several failure planes for the LSF of interest to address the record-to-record variation of earthquakes to consider its stochastic nature. This needs special consideration for the development of an efficient adaptive SVR approach for SeRA of structures.

The present study proposed an adaptive metamodeling approach of SeRA where the metamodels are constructed directly avoiding the distribution assumption. In doing so, the SVR model is chosen as the metamodel considering its generalization capability with better accuracy that can circumvent overfitting compared to empirical risk minimization-based metamodeling methods, e.g., polynomial RSM and ANN (Shyamal Ghosh, Ghosh, and Chakraborty 2018). Unlike the dual RSM, the present study proposes a direct approach where separate SVR models are constructed for the approximation of response for each ground motion in the considered ground motion bin. This was explored previously by using Kriging (Shyamal Ghosh, Roy, and Chakraborty 2021) and SVR (Shyamal Ghosh, Roy, and Chakraborty 2018) based metamodels for SeRA. However, the metamodels in those studies were trained by a one-shot DOE. It is already discussed that the adaptive sampling approach to construct a metamodel for static SRA is well-versed to ensure better accuracy of reliability analysis. However, no such adaptive sampling-based metamodeling approach for the SeRA of structure is studied where the metamodels are constructed directly without the distribution assumption of responses. Thus, the present study is expected to contribute significantly to SeRA as it proposes an innovative adaptive sampling strategy with better efficiency than the existing approaches (Yanjie Xiao, Yue, and Zhang 2021; Yanjie Xiao et al. 2022; Jeddi et al. 2022). The proposed approach updates the training samples for each SVR model from an initial DOE to obtain reliabilities for different seismic intensities. This involves considerations of separate failure surfaces for each LSF at a given intensity level. Moreover, the variability in the locations of the failure planes due to record-to-record variations of ground motions is addressed appropriately. In detail, separate reduced spaces for each ground motion in the bin are constructed in the proposed adaptive scheme for each seismic intensity. For this, a cross-validation-based error norm is employed to identify the candidate samples that have a high chance of being predicted in the wrong domain (safe sample in failure domain or vice-versa). From such a reduced space, the sample located at the maximum distance from its nearest training sample is selected as the new training sample. This goes on iteratively until the failure estimate stabilizes. The effectiveness of the proposed approach is elucidated by considering three numerical examples considering the MCS-based SeRA results as the benchmark for comparative study. Furthermore, results are also compared with the state-of-the-art active learning-based SeRA approach (Yanjie Xiao, Yue, and Zhang 2021).

2. Metamodeling-based seismic reliability analysis of structures

Unlike SRA under static or deterministic dynamic loads, the application of the metamodeling approach for SeRA is a difficult task. The reason is that it will involve a large number of input parameters due to the high-dimensional nature of earthquake input force. The dual RSM is generally applied to circumvent this difficulty (Lin and Tu 1995). A suite of ground motions is considered so that the effect of record-to-record variations can be implicitly included in the analysis. At each DOE point, the values of any desired response at a specific seismic intensity level are evaluated for all the input ground motions in the suite to compute the mean value of the desired response quantity, μ_Y and its standard deviation (SD), σ_Y. Now, to approximate the mean and SD of the response, the metamodels for the mean response $\hat{g}_{\mu}(\mathbf{x})$ and its SD, $\hat{g}_{\sigma}(\mathbf{x})$ are constructed for predicting these quantities at any combination of structural parameters, $\mathbf{x}$. Finally, the response quantity of interest is obtained based on the assumption that the overall response follows a lognormal probability density function (PDF).

Instead of the dual RSM approach as briefed above, the present study constructs the metamodels directly for each ground motion in the bin for response approximation. This will be advantageous because, unlike the dual RSM approach, the lognormal assumption for overall response approximation is not necessary. A similar approach was also explored by using Kriging (Shyamal Ghosh, Roy, and Chakraborty 2021) and SVR (Shyamal Ghosh, Roy, and Chakraborty 2018) methods based on the training data obtained by a one-shot DOE approach. The metamodel $\hat{g}_k(\mathbf{x})$ for approximating the responses of the *k*-th ground motion (y_k) can be obtained as,

$$y_k = \hat{g}_k(\mathbf{x}), \qquad k = 1, 2, \ \ldots.., m \tag{2}$$

where **x** represents the vector of structural parameters, *m* is the total number of ground motions considered in the bin. The conventional notion to consider the record-to-record variation to reflect the stochastic nature of earthquake motion is implicitly incorporated by random selection of metamodel. Note that the seismic response approximation does not require a prior assumption on its distribution. Further, the peak ground acceleration (PGA), considered as the earthquake intensity parameter, is also taken as one of the predictors in the metamodel. Thereby, Eq. (2) is rewritten as,

$$y_k = \hat{g}_k(\mathbf{x}, \mathrm{PGA}) = \hat{g}_k(\mathbf{X}) \tag{3}$$

The input vector **X** in the above comprises structural parameters (**x**) as well as the PGA values.

3. Proposed Adaptive SVR-based MCS framework for seismic reliability analysis

The present study explores an adaptive metamodeling approach for SeRA of structures, where the metamodels are constructed directly. It avoids the distribution assumption as is necessary for the usual dual metamodel-based approach (Towashiraporn 2004). Specifically, an adaptive SVR approach for the SeRA of structures in the MCS framework is attempted and presented in this section.

3.1 Support vector regression

The SVR-based metamodeling approach is first briefed in this section as the present study hinges on the SVR-based metamodel. The SVR is based on the principle of structural risk minimization of statistical learning theory (Vapnik 1995; 1998). The SVR can be applied for both linear and nonlinear regressions. For the sake of a concise presentation, the mathematical details are skipped here. However, the basic concept of SVR is briefly discussed in the Appendix. More details can be found in Smola and Schölkopf (Smola and Schölkopf 2004). The SVR can be readily implemented in MATLAB using the Gunn toolbox available at http://www.isis.ecs.soton.ac.uk/resources/svminfo/. The fitting of an SVR model based on the ε-insensitive loss function involves a regularization parameter, *C*, apart from the loss function parameter ε. These parameters control the complexity and degree to which deviations larger than a specified value are tolerated. Further, the Gaussian radial basis function (GRBF) adopted as kernel function in the presented study to develop SVR-based metamodel to tackle nonlinear response approximation involves another unknown parameter σ i.e., the extent of the GRBF kernel also has a significant effect on the training process. The optimum choices of these parameters can be decided by cross-validation techniques. A simple yet effective algorithm proposed by Roy, Manna, and Chakraborty (2019) is applied to obtain the values of *C*, ε and σ to construct the SVR model. The algorithm solves an optimization sub-problem to minimize the generalized root mean square error (GRMSE) value obtained by the cross-validation method. The GRMSE can be defined as,

$$GRMSE = \sqrt{\frac{1}{p}\sum_{i=1}^{p}\left(f_i - \hat{f}_i^{\,i-1} \right)^2} \tag{4}$$

where, $\hat{f}_i^{\,i-1}$ represents the prediction at the *i*-th sample point based on the metamodel constructed using all the sample points in the DOE except the *i*-th sample point, p is the total number of sample points in the DOE and f_i is the actual response at the *i*-th sample point.

3.2 Adaptive sampling scheme of the proposed approach

An initial DOE is selected first to start the proposed adaptive SVR-based metamodeling approach of SeRA of structures. Based on the initial DOE, separate SVR models are constructed for each ground motion. The initial SVR model for a particular ground motion is identical for different seismic intensities. However, as the adaptive SVR modelling process progresses, separate models are constructed for each seismic intensity. Consequently, after undergoing individual active-learning processes, these final adaptive SVR models do exhibit variations corresponding to different seismic intensities. Therefore, separate reduced spaces are to be constructed to obtain adaptive training samples to construct the SVR model iteratively. The stopping condition of updating each adaptive SVR metamodel also needs to be judged separately. The algorithm is presented in the following sub-sections.

3.2.1 Initial DOE and initial SVR metamodels

In SeRA problems involving implicit LSF, the position of the failure plane is not known a priori. An initial DOE should be constructed using samples that are distributed as uniformly as possible over the entire input space until an approximated location of the failure boundary is obtained. This can be achieved by a space-filling design (Santner, Williams, and Notz 2003) that is suitable for computer experiments where replication error is absent, unlike physical experiments. The uniform design (UD) (Fang et al. 2000) is preferred over other space-filling designs for its minimum discrepancy from the theoretical uniform distribution. In order to construct an initial DOE, a number of training samples (say, p_0) are selected following a UD over the entire physical domain of the input variables. The physical domain for PGA is considered as 0.1 g to 1.0 g. The ground motion bin consists of m numbers of scaled earthquake time histories. Now, the responses are evaluated at p_0 sample points using each earthquake time history to construct the corresponding SVR-based response prediction model. Thus, total m numbers of initial SVR models are constructed for the m numbers of ground motion records in the bin. As discussed earlier, each SVR model is built by optimizing an ε-insensitive loss function. The hyperparameter tuning algorithm proposed by Roy, Manna, and Chakraborty (2019) is applied here by replacing the leave-one-out cross-validation method with the holdout cross-validation method. In the holdout cross-validation method, one-third of the samples of the DOE having lesser LSF magnitude are held out for the test set and the remaining samples as the training set. The leave-one-out cross-validation method approximately takes p (number of data points) times higher computational time than the holdout cross-validation method. Thus, the holdout cross-validation method instead of the leave-one-out cross-validation method is employed for efficiency. The minimum GRMSE value is noted for each of the m numbers of SVR models, i.e., $e_{GRMSE,1}^{\min}, e_{GRMSE,2}^{\min}, \ldots, e_{GRMSE,m}^{\min}$ are the m numbers of minimum GRMSE values noted sequentially for each of the m metamodels corresponding to each of the ground motions in the bin.

3.2.2 Reduced space and adaptive SVR metamodels

For a particular SVR model, reduced spaces are constructed with samples having a predicted magnitude of LSF less than the corresponding noted minimum GRMSE value. For example, for a certain value of PGA, a set of MCS points, $\boldsymbol{\Omega}_k$ is identified as the reduced space for the k-th SVR model corresponding to the k-th ground motion record as,

$$\Omega_k = \left\{ \mathbf{X} \,|\, \left|\hat{g}_k(\mathbf{X})\right| < e_{GRMSE,k}^{\min} \right\} \tag{5}$$

where $\mathbf{X}$ represents an MCS sample point that is randomly assigned to the *k*-th ground motion, $\left|\hat{g}_k(\mathbf{X})\right|$ is the magnitude of the approximated LSF at that point, and $e_{GRMSE,k}^{\min}$ represents the noted minimum GRMSE value for the *k*-th ground motion. Then, a new training sample is selected from the set Ω_k by the maximin distance criterion (Johnson, Moore, and Ylvisaker 1990) to avoid data clustering. The actual response is evaluated at the selected point for the *k*-th ground motion record and the selected value of PGA. By including the new training data into the existing DOE for the *k*-th ground motion record and the selected value of PGA, the associated SVR model is updated. Then, the SVR model hyperparameters are obtained by the holdout cross-validation-based approach as described in the previous sub-section. Subsequently, the $e_{GRMSE,k}^{\min}$ value is updated. Then, the set Ω_k is reconstructed following Eq. (5). Again, a new training sample selected by the maximin distance criterion from the updated reduced space is added to the adaptive DOE to update the SVR model. The updating of the SVR model is iteratively continued until a convergence criterion is satisfied. For MCS points randomly assigned to the *k*-th ground motion record, the number of failures at two consecutive iterations obtained by the adaptive SVR models are considered for the stopping condition. The convergence criterion in the present study is taken as,

$$\left|\hat{n}_{f,i}^k - \hat{n}_{f,i-1}^k\right| \big/ \hat{n}_{f,i}^k \le 0.05 \tag{6}$$

where, $\hat{n}_{f,i-1}^k$ and $\hat{n}_{f,i}^k$ are the values of the number of failures at ($i-1$) and *i*-th iterations, respectively. The threshold for relative change of reliability estimate between two consecutive iterations can be typically chosen between 10^{-4} and 10^{-1} (Wong, Hobbs, and Onof 2005). This threshold range is also recommended for the stopping condition of the adaptive SVR method (Dai et al. 2012). The value of 0.05 is within the range and is effective on a similar adaptive SVR approach for SRA (Roy and Chakraborty 2022). Besides this stopping condition, if there is no new sample left in the updated set Ω_k to add to the adaptive DOE, then iteration also stops.

3.3 Outline of the proposed adaptive SVR approach

The values of P_f are required to obtain at various PGA values for seismic vulnerability assessment of structures. Thus, to obtain P_f values, an MCS population of N_{MC} points consisting of randomly generated samples of the structural parameters (**x**) is considered. The control variable PGA is kept constant, for which P_f needs to be calculated. Then, each of the N_{MC} points is randomly assigned to an earthquake time history from the bin of m number of ground motion records. Without loss of generality, it is assumed that N_k points are assigned to the k-th ground motion. The step-by-step procedure of the proposed adaptive SVR approach to approximate the seismic response corresponding to the k-th ground motion at a certain value of PGA is as follows:

Step 1 – Select an initial DOE with p_0 number of training samples by a space-filling design over the entire input space.

Step 2 – Obtain the SVR hyperparameters by minimizing the GRMSE value obtained by the holdout cross-validation approach. Note the minimum GRMSE value.

Step 3 – Obtain the approximate values of the LSF at the N_k points from the SVR model. Estimate the number of failure points based on the SVR model.

Step 4 – Built the reduced space following Eq. (5) with the noted GRMSE and the predicted LSF values.

Step 5 - To add one new training sample, select a point from the reduced space by the maximin distance criterion. If no new training point is left in the reduced space, go to step 9.

Step 6 – Update the SVR model by adding the selected new training sample to the DOE.

Step 7 – Obtain the number of failure points based on the updated SVR model.

Step 8 - If the number of failure points in the previous iteration is within $\pm$5% of that of the present iteration, then updating of the SVR model is stopped; otherwise, go to step 2.

Step 9 - The latest value of the number of failure points is considered as the converged result (say, $\hat{n}^k_{f,converged}$).

Thus, starting from a common initial DOE, *m* numbers of adaptive SVR models for *m* number of ground motion records are updated until the convergence. The probability of failures at the desired value of PGA is finally estimated by the proposed SVR approach in the MCS framework as,

$$\hat{P}^{SVR}_{f,PGA} = \frac{\sum_{k=1}^{m} \hat{n}^k_{f,converged}}{\sum_{k=1}^{m} N_k} = \frac{\sum_{k=1}^{m} \hat{n}^k_{f,converged}}{N_{MC}} \tag{7}$$

For different PGA values, the iterations start from the same initial DOE. However, the reduced spaces and adaptive samples are different. In the case of multiple damage levels, the initial DOE is also identical for all LSFs but reduced spaces are different. Thus, different new training points are added. Hence, separate adaptive SVR models are constructed. A flowchart of the proposed adaptive SVR approach for SeRA is shown in Figure 1.

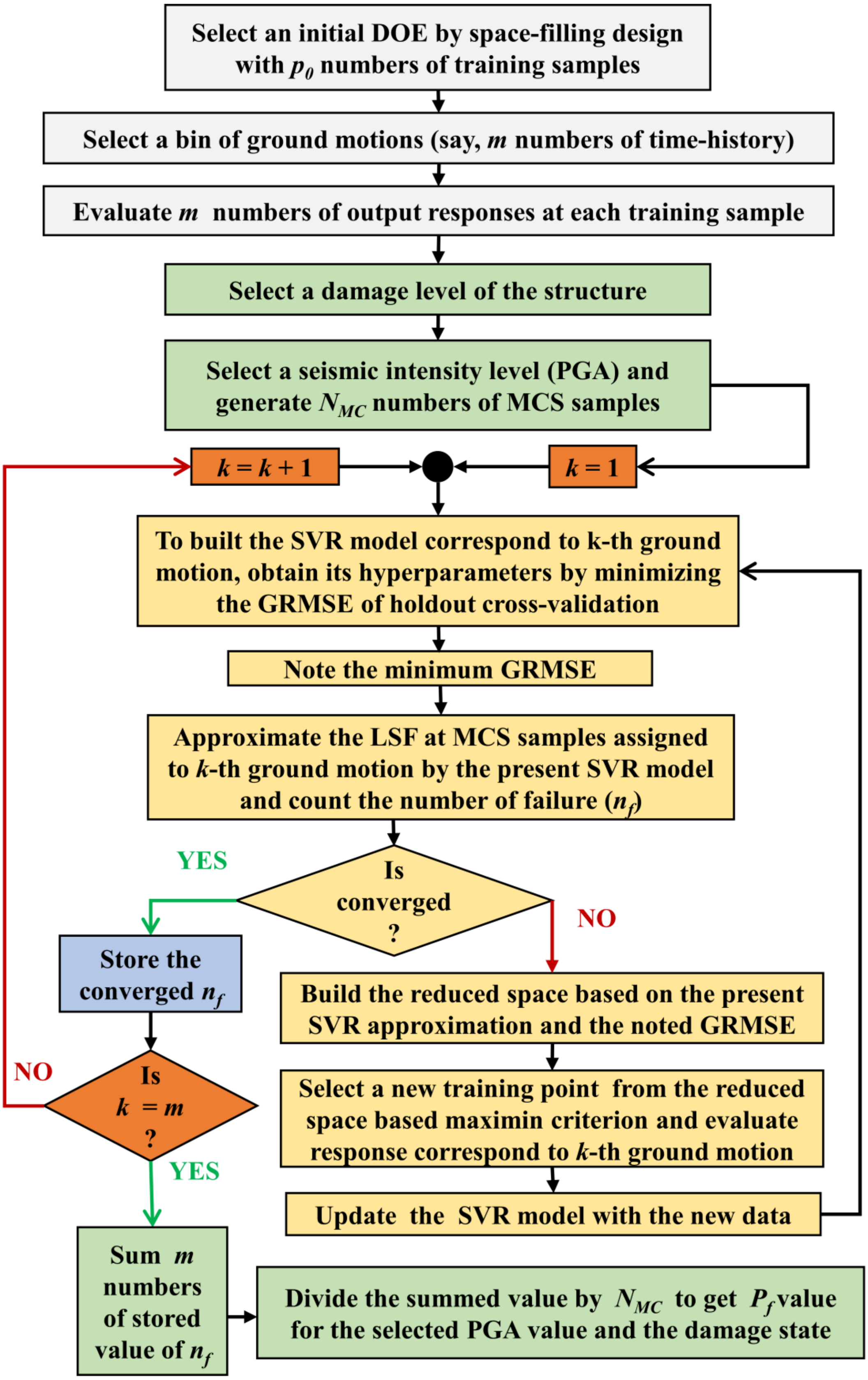

Figure 1 The flowchart of the proposed adaptive SVR approach of SeRA.

5. Numerical Study

The effectiveness of the proposed adaptive SVR approach for SeRA of structure is elucidated numerically by considering three examples. The first example is a simple nonlinear single-degree of freedom (SDOF) system. Because of its simplicity, it is feasible with reasonable time to obtain a large number of nonlinear responses of the SDOF system with random system properties and earthquake inputs necessary for reliability computation by brute force MCS. Thus, this problem has been taken up to make a comparative study of the accuracy possible to achieve in SeRA by the proposed approach compared to that obtained by the brute force MCS technique. The other two examples are more realistic, i.e., a typical bridge pier of a multi-span simply supported river bridge and a four-storied reinforced concrete building frame considered to be located in the Guwahati city of India. The second example is a comparatively higher-dimensional problem having eight random variables. The ground motion bin consists of eight recorded accelerograms, eight artificial accelerograms consistent with the design spectrum of the considered location and the rest eight are synthetically generated for Guwahati city (Shyamal Ghosh, Ghosh, and Chakraborty 2018) and readily available in Swarup Ghosh (2020). The same 24 earthquake time histories (eight recorded, eight artificial, and eight synthetic) are considered for all the example problems of the present study.

5.1 Example 1: A nonlinear single degree of freedom system

The nonlinear SDOF, as shown in Figure 2 (a), is considered as the first example to illustrate the proposed adaptive SVR approach for SeRA. It is characterized by a nonlinear spring connecting a lumped mass. The nonlinear force-deformation (F-u) behaviour of the spring is defined in Figure 2 (b). The mass and stiffness proportional damping is considered for the dynamic analysis. The frequency (ω), damping (ξ), yield force (F_y) and the ratio of post-yield to elastic stiffness of the nonlinear spring (α) are considered random variables. The statistical properties of the random variables assumed to be truncated Gaussian are depicted in Table 1. The PGA value is considered as the control variable whose range varies from 0.1 g to 1.0 g. A bin of 24 ground motions, as mentioned at the beginning of the numerical study section, is considered. The maximum displacement of the mass is taken as the output response variable. Two different damage states (slight and complete damages) with respect to two different threshold displacement values (0.4 m and 0.8 m) are considered to demonstrate the proposed SeRA approach.

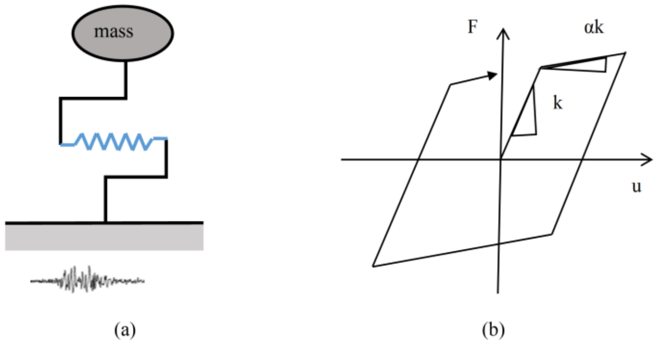

Figure 2 (a) The SDOF system and (b) the force-deformation behaviour of the nonlinear spring.

Table 1 The statistical properties of the random variables of the SDOF system

Random variables	Unit	Mean	Coefficient of variation	Truncation limits	
				Upper	Lower
ω	rad/s	6.28	0.2	9.27	3.29
ξ	-	0.02	0.25	0.03	0.01
F_y	N	1.974	0.2	2.913	1.035
α	-	0.05	0.25	0.075	0.025

An initial DOE is constructed first following UD over the entire physical domain of the input variables to implement the proposed adaptive SVR approach. The training points of the initial DOE are taken within the range of all the five variables (the four random variables and PGA) by arranging 30 equidistant levels of each variable according to the UD table, $U_{30}(30^5)$ readily available at https://www.math.hkbu.edu.hk/UniformDesign/. It implies that 30×24 = 720 responses are evaluated to build 24 initial SVR models. Now for different PGA values, the reliability is estimated by the proposed adaptive SVR approach for both the damage states. The value of N_{MC} is taken as 30000 to apply the proposed adaptive SVR approach. For each PGA value, 24 different adaptive SVR models are constructed for a particular damage state. The procedure of obtaining the SVR models for each intensity of earthquakes and each ground motion in the considered bin is generic in nature. However, the underlying details, e.g., the number of iterations and the total samples added to the DOE to obtain the adaptive SVR model for each ground motion could be different. The maximum number of iterations required by the proposed adaptive SVR approach to obtain the reliability of the system for each PGA value in cases of slight and complete damage states are shown in Figures 3 (a) and (b), respectively. Further, the performance of the proposed approach is studied with different stopping thresholds (i.e., 5%, 1%, and 10%). It is observed that the number of maximum iterations involved for different cases is quite different and does not follow any trends.

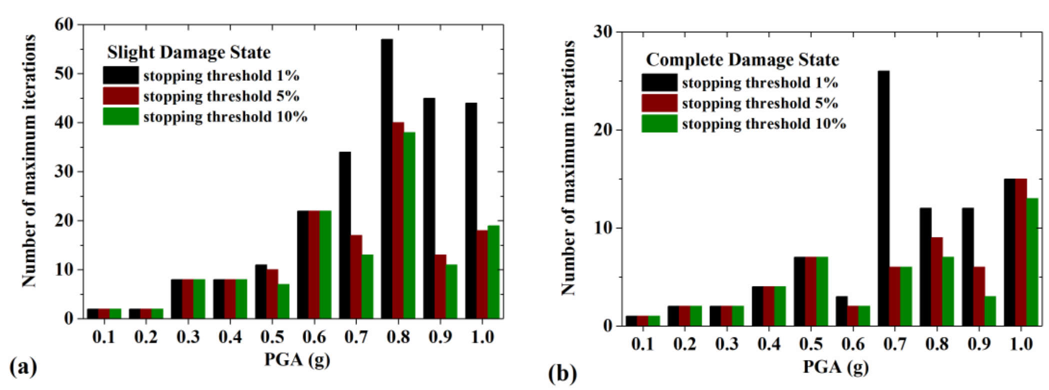

Figure 3 Numbers of maximum iterations required by the proposed adaptive SVR approach for SeRA of the SDOF system for varying PGA at (a) slight and (b) complete damage states.

The total number of adaptive samples required for a damage state at a particular PGA level is further noted. The numbers of adaptive samples added by the proposed approach for SeRA of the SDOF system for ten different PGA values for slight and complete damage states are shown in Figures 4 (a) and (b), respectively. In addition, the absolute errors in estimating the failure probabilities for each case are compared in Figures 5 (a) and (b) for three different stopping thresholds. As expected, the number of maximum iterations and the number of adaptive samples required decreases with the relaxation of the threshold. However, an increased number of iterations does not always provide better accuracy. It is generally expected that a tighter stopping threshold yields a smaller error. However, reliability results oscillate during the iteration process and sometimes results cross the zero-error line i.e., from the positive error side to the negative error side or vice-versa. In this regard, tighter stopping thresholds guarantee only a lower amount of oscillation, not smaller errors. Therefore, a relaxed stopping threshold (e.g., 10%) can stop the iteration early when the oscillation magnitude is high, and there is always a chance that it may stop when it is very close to the zero-error line. This happens in a few cases of complete damage state. Based on the observation of the overall performance, the 5% threshold is found to be an optimum trade-off between accuracy and computational demand. The total number of adaptive samples added to estimate the P_f values corresponding to the ten different PGA values are 494 for the slight damage case and 94 for the complete damage case. Thus, a total of 588 numbers of adaptive samples is required. Thereby, the total number of actual response evaluations required by the proposed adaptive SVR approach is 720 + 588 = 1308 for the estimation of reliabilities for ten different PGA values.

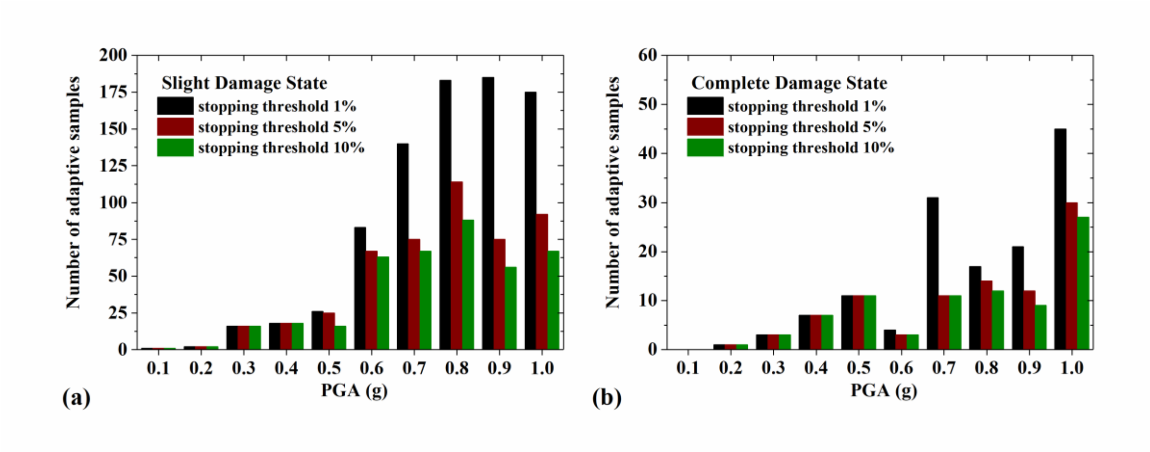

Figure 4 Numbers of adaptive samples added by the proposed adaptive SVR approach for SeRA of the SDOF system with varying PGA at (a) slight and (b) complete damage states.

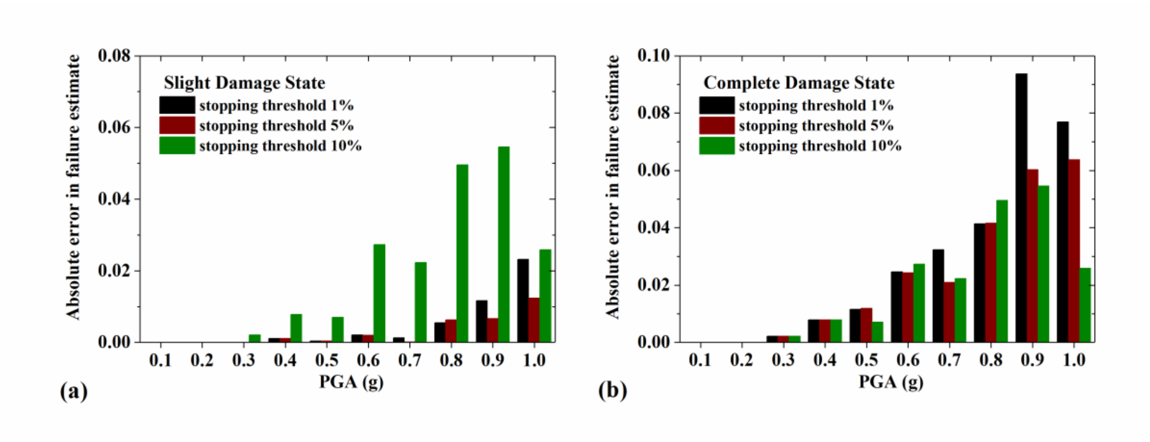

Figure 5 Absolute deviation in estimation of seismic reliability of the SDOF system with varying PGA at (a) slight and (b) complete damage states.

To study the efficiency of the proposed adaptive SVR-based approach in estimating seismic reliability, a comparative study is further made with the reliability results obtained by the state-of-the-art active learning approach for SeRA (Yanjie Xiao, Yue, and Zhang 2021) in combination with the Kriging-based metamodel considering the direct MCS-based reliability results as the benchmark. The active learning approach for SeRA (Yanjie Xiao, Yue, and Zhang 2021) employs the expected feasibility learning function and if the maximum expected feasibility function value of candidate points is less than a specified tolerance (e.g., a value of approximately 1/20 of the SD of the function values at the initial training samples), the metamodel is considered to have sufficient accuracy and the sampling can be stopped. For SeRA by the direct MCS, the simulation is performed for any desired PGA level. The random structural parameters are simulated corresponding to their respective PDF and are combined at random to generate a large number (thirty thousand herein) of the SDOF system. The maximum displacement is obtained for each of such SDOF systems by randomly selecting a ground motion from the bin (following the assumption that each earthquake is equally likely to occur) for the considered seismic intensity level. The same procedure is repeated for all the simulated samples of the SDOF system. The probability of exceeding a given threshold displacement is obtained accordingly from the ensemble yielding the probability of failure for the considered level of seismic intensity. The results obtained by the direct MCS (denoted as DMCS), the proposed adaptive SVR approach and the mentioned active learning-based Kriging approach (denoted as Active Kriging) are compared for slight and complete damage states in Figures 6 (a) and (b), respectively. The improved performance of the proposed adaptive SVR approach over the active Kriging approach for most of the PGA levels for both the damage states can be readily observed from these plots. The computational time for each method is reported in Table 2. However, the computation time of a method may vary for different CPU configurations. In this regard, reporting the number of function evaluations seems to be a better alternative for comparing the computational cost as it is independent of the execution platform (e.g., CPU configuration, coding software, etc.). Thus, the number of function evaluations required is also reported in addition to computation time. The number of function evaluations (therefore also the computation time) for obtaining the common initial DOE for each PGA should not be over-counted. Therefore, this computation cost is counted only once for calculating the total computation cost of metamodeling approaches to get the complete reliability curve. In the case of the direct MCS technique, the reliability results are noted to be converged at 30000 simulations. Thus, 30000 actual function evaluations are involved for each PGA; thereby, 3×10^5 actual function evaluations for all the ten PGA values are needed to get the complete reliability curve. Whereas a total of 1308 and 104448 actual function evaluations are required for SeRA by the proposed adaptive SVR approach and the active Kriging approach, respectively. The computation time of the proposed adaptive SVR approach is observed to be the least for all cases. This clearly revealed the effectiveness of the proposed approach.

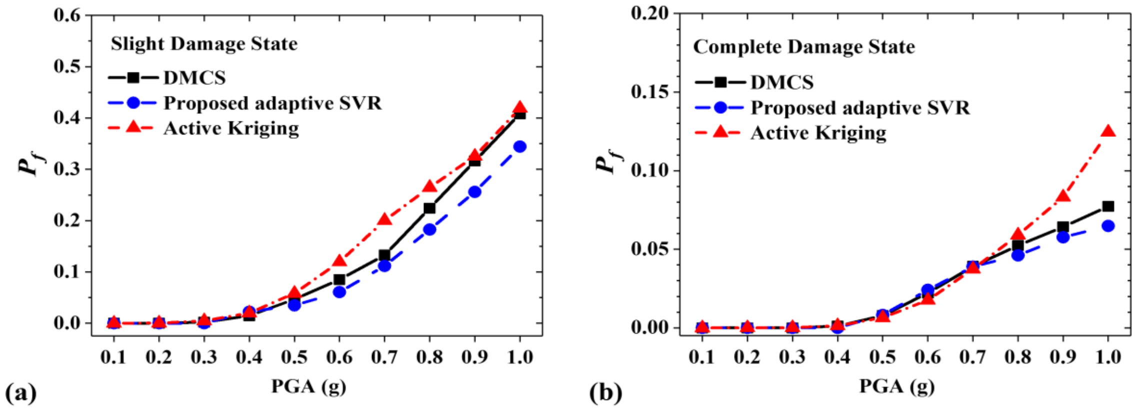

Figure 6 The comparisons of seismic reliability of the SDOF system with varying PGA at (a) slight and (b) complete damage states.

Table 2 Comparison of computation time and required number of actual function evaluations for SeRA of the SDOF system by different approaches

PGA	Proposed adaptive SVR		Active Kriging		DMCS	
	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations
0.1 g	1045	721	525	1920	1749	30000
0.2 g	1070	723	556	2304	1750	30000
0.3 g	1271	739	873	5280	1760	30000
0.4 g	1346	745	2682	13296	1804	30000
0.5 g	1597	765	2953	13896	1789	30000
0.6 g	1936	792	3518	15240	1785	30000
0.7 g	2112	806	3535	15264	1812	30000
0.8 g	2640	848	3219	14544	1824	30000
0.9 g	2125	807	3348	14784	1780	30000
1.0 g	2564	842	3187	14400	1751	30000
Total	13817	1308	20507	104448	17804	300000

5.2 Example 2: A multi-span simply supported bridge pier

A critical pier of a multi-span simply supported river bridge, considered to be located in the Guwahati city of India, is taken as the next example problem. The longitudinal profile of the bridge is shown in Figure 7 (a). The details geometry of the bridge, the bent columns, the supporting bent cap, and the pile caps are given in Shyamal Ghosh, Ghosh, and Chakraborty (2018). The concrete and steel grades are considered as M25 (characteristic compressive strength of 25 N/mm^2) and Fe 250 (yield strength of 250 N/mm^2), respectively. The elevation of the pier and its OpenSees model are shown in Figures 7 (b) and (c), respectively. The NDSRA is performed in the OpenSees software (McKenna et al. 2016). Further details about the displacement-based beam-column elements with associated fibre sections for the bent cap and columns with associated concrete (cover and core separately), modelling of reinforcement and that of pile resistance including soil-structure interaction effects can be seen in Shyamal Ghosh, Ghosh, and Chakraborty (2018).

The random variables considered for SeRA are the characteristic compressive strength of concrete (f_{ck}), the elastic modulus of concrete (E_c), the yield strength of steel (f_y), the elastic modulus of steel (E_s), translational spring constants ($K_{G,h}$), rotational spring constants ($K_{G,r}$) and the structural damping (ξ). The properties of these variables are shown in Table 3. The same ground motion bin consists of 24 earthquake time histories considered in the previous example is taken for NDSRA. The permissible drift ratio (i.e. the maximum displacement of the pier divided by its height) associated with slight, moderate, extensive and complete damage states are taken as 0.01, 0.025, 0.05 and 0.075, respectively (Kim and Feng 2003).

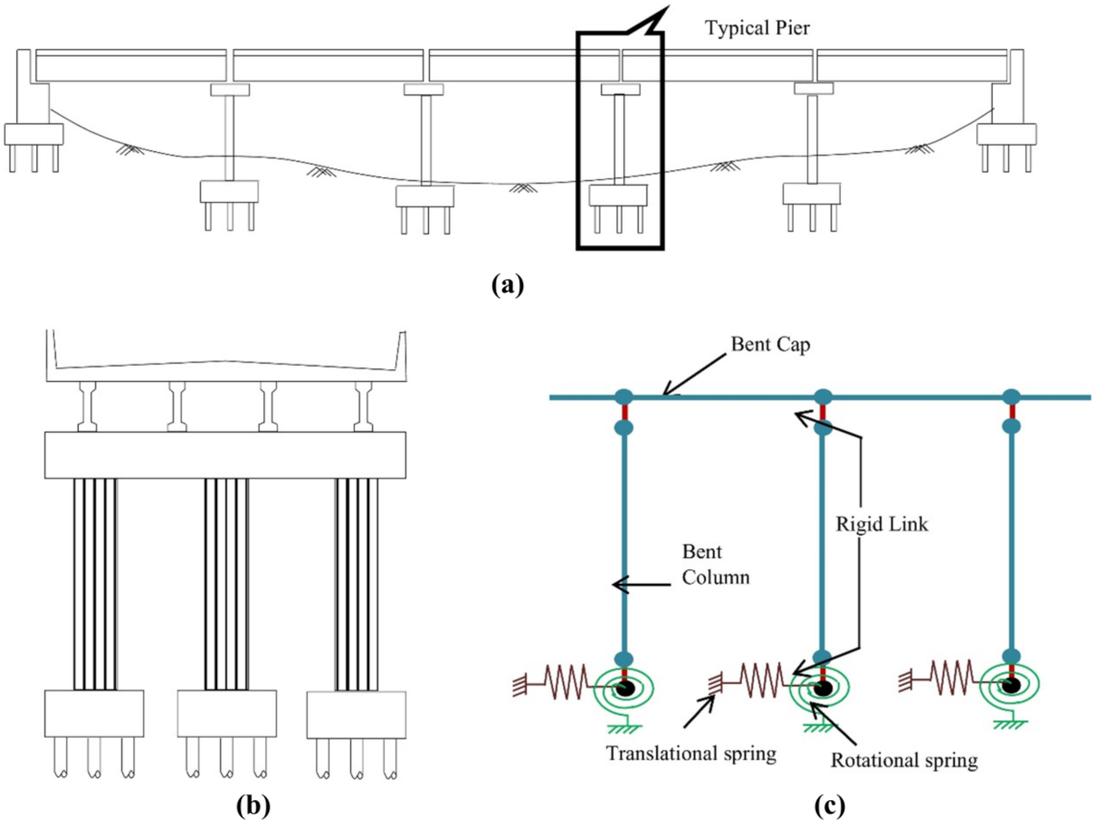

Figure 7 The details of the considered bridge: (a) the longitudinal profile of the bridge, (b) the elevation of the considered multi-column bent and (c) the OpenSees FE model of the bent (Shyamal Ghosh, Roy, and Chakraborty 2018).

Table 3 Statistical properties of the random variables of the bridge pier.

Random variables	Unit	Distribution type	Mean	Coefficient of variation	Truncation limits	
					Lower	Upper
f_{ck}	MPa	Gaussian	35	0.064	32.76	37.24
E_c	MPa	Lognormal	29580	0.077	27302.34	31857.66
f_y	MPa	Gaussian	500	0.064	468	532
E_s	MPa	Lognormal	2×10^7	0.08	1.84×10^7	2.16×10^7
$K_{G,h}$	kN/mm	Uniform	130.5	0.289	65.25	195.75
$K_{G,r}$	kN-m/rad	Uniform	6.06×10^5	0.289	3.03×10^5	9.09×10^5
ξ	-	Gaussian	0.045	0.278	0.0325	0.0575

According to the UD table, $U_{30}(30^8)$ readily available at https://www.math.hkbu.edu.hk/UniformDesign/, 30 training points are considered within the range of all the eight variables by arranging 30 equidistant levels of each variable to construct an initial DOE. Then, 30×24 = 720 responses are evaluated, and 24 initial SVR models are constructed. Now, P_f is obtained by the proposed adaptive SVR approach for four different damage states for different values of PGA. The number of maximum iterations required for ten different PGA values is shown in Figure 8. Like the previous example, it is observed that these numbers do not have any trend. Furthermore, the numbers of adaptive samples added by the proposed adaptive SVR approach for each PGA value and for each damage state are shown in Figure 9. The number of adaptive samples added to obtain P_f values for all the ten PGA values are 114, 88, 104 and 50 for slight, moderate, extensive and complete damage states, respectively. The total number of adaptive samples to obtain the final SVR model is 114 + 88 + 104 + 50 = 356. Thus, the number of actual response evaluations by the proposed adaptive SVR approach is 720 + 356 = 1076 for this problem.

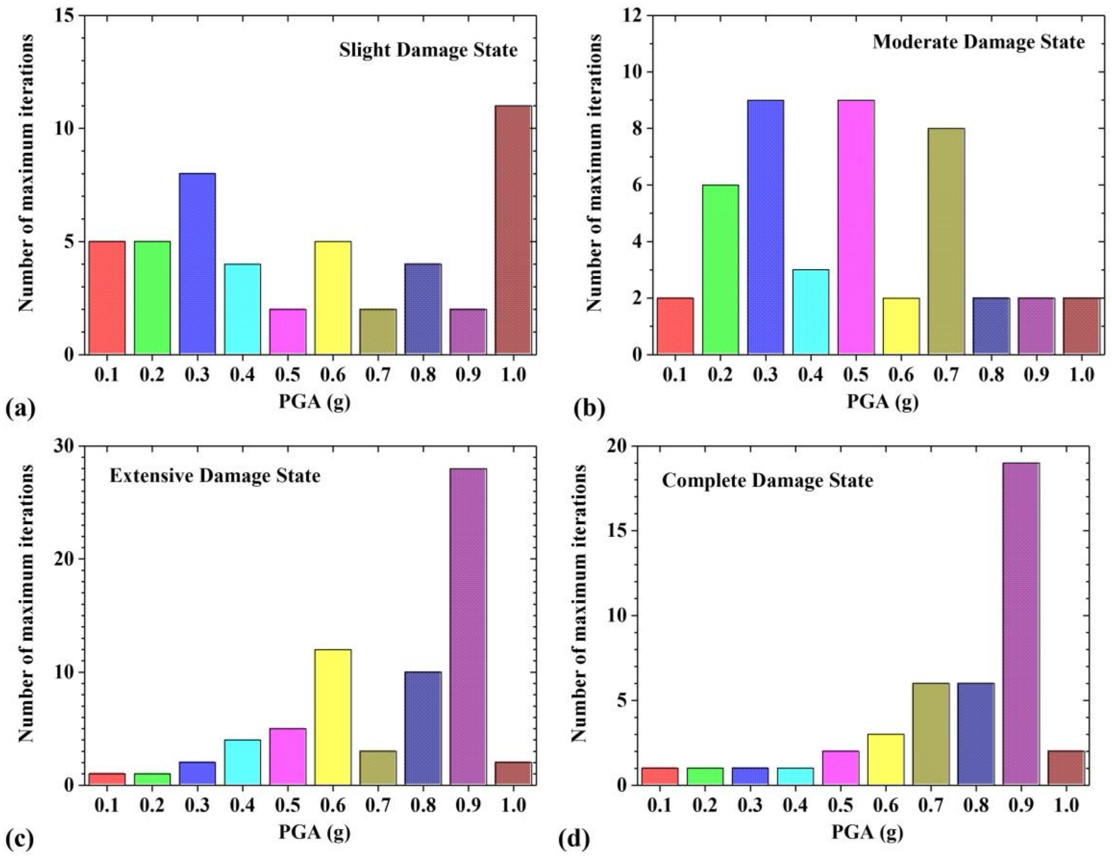

Figure 8 Numbers of maximum iterations required by the proposed adaptive SVR approach for SeRA of the bridge pier with varying PGA at (a) slight, (b) moderate, (c) extensive and (d) complete damage states.

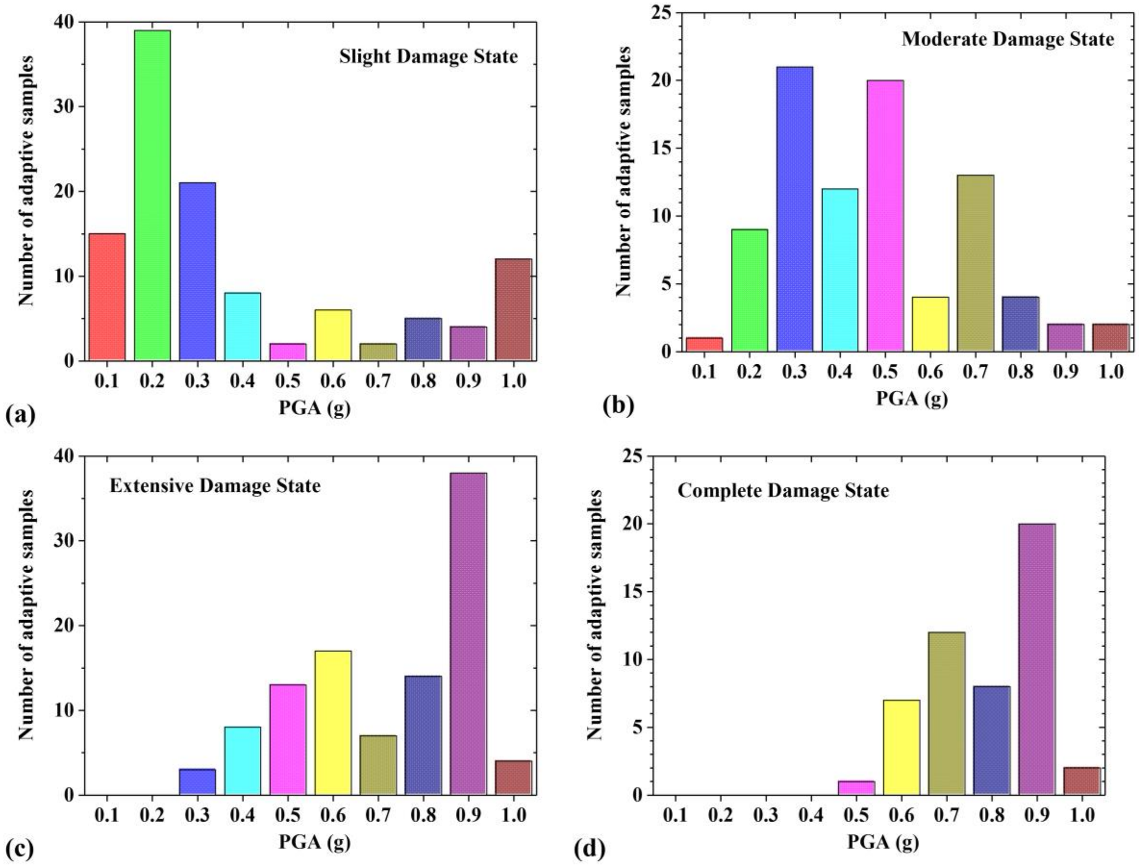

Figure 9 Numbers of adaptive samples added by the proposed adaptive SVR approach for SeRA of the bridge pier with varying PGA at (a) slight, (b) moderate, (c) extensive and (d) complete damage states.

For estimating P_f values by the direct MCS method, a limited number of simulations (5000 MCS samples for each PGA value) study is performed to get the trend of the brute-force MCS-based solution so that the quality of seismic reliability results can be judged for different approaches. It may be noted that with 5000 simulations, the P_f value obtained cannot be claimed as the final converged value. The number of simulations required could be more than 5000 for getting the final converged P_f value by the brute-force MCS. However, this needs enormous computation time. Even, for such limited simulation, a total of 5000×10 = 50000 simulations (approximately, 4 to 5 s for each simulation resulting in 206933 s or 57.5 h of computation time on a CPU with Intel i5-3570 3.40 GHz processor and 16 GB RAM) are required. It is realized that the P_f values obtained based on 5000 simulations give the trend of the failure probability results and will be helpful to judge the quality of reliability results obtained by the metamodeling approaches. Thus, to study the nature of the variation of P_f, the variation of P_f with the number of simulations is shown in Figure 10 for PGA of 0.5 g for moderate and extensive damage states.

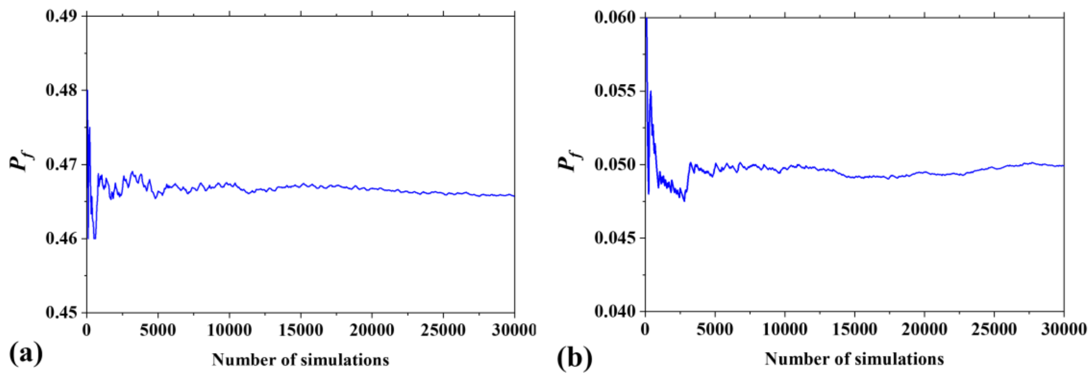

Figure 10 Convergence study of failure probability at 0.5 g PGA for (a) moderate and (b) extensive damage states.

Like the previous example, the P_f values are also obtained by the active learning-based Kriging approach. The P_f values obtained by different approaches are compared in Figure 11 for four damage states with varying PGA. In most of the cases, the reliability results obtained by the proposed adaptive SVR approach are much closer to the results of the direct MCS. At the same time, the results of the active learning-based Kriging approach in many cases are far away from the reference results. On the other hand, the proposed adaptive SVR approach performs well for any PGA value for all damage states. Thus, it is expected to be a better choice. Further, the computation time and the number of function evaluations required for each method are compared in Table 4. It may be noted that the direct MCS method, even with limited MCS study, 5000 actual function evaluations for each PGA and 50000 actual function evaluations for all PGA is needed to develop the reliability curve. Whereas a total of 1076 actual function evaluations are required by the proposed adaptive

SVR approach for SeRA. On the contrary, 37800 actual function evaluations are required by the active Kriging. In this regard, it is to be noted that the active learning-based Kriging approach requires 24 NDSRA for adding one new training sample and this is the reason for a larger number of required function evaluations.

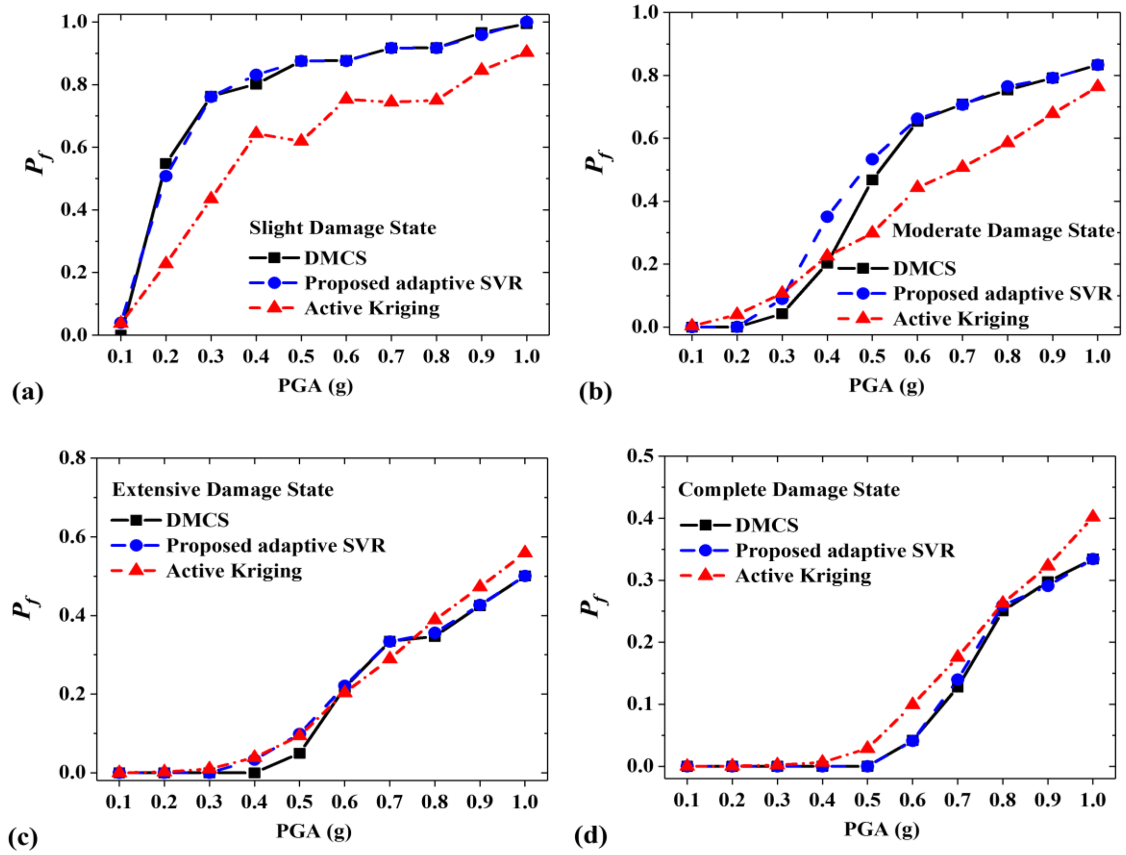

Figure 11 Comparisons of seismic reliability of the bridge pier with varying PGA at (a) slight, (b) moderate, (c) extensive and (d) complete damage states.

Table 4 Comparison of computation time and required number of actual function evaluations for SeRA of the bridge pier by different approaches

PGA	Proposed adaptive SVR		Active Kriging		DMCS	
	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations
0.1 g	4526	736	14219	4128	17573	5000
0.2 g	5058	768	15071	4224	19560	5000
0.3 g	5024	765	15847	4272	21284	5000
0.4 g	4737	748	16327	4344	20851	5000
0.5 g	4882	756	17034	4536	22555	5000
0.6 g	4846	754	17646	4800	22267	5000
0.7 g	4830	754	18141	4992	19788	5000
0.8 g	4790	751	16577	4584	21421	5000
0.9 g	5336	784	15892	4416	20757	5000
1.0 g	4604	740	14351	3984	20877	5000
Total	21003	1076	133477	37800	206933	50000

5.3 Example 3: a four-storied building frame

A four-storied reinforced concrete building frame previously studied by Shyamal Ghosh, Roy, and Chakraborty (2018) is taken as the next example problem. The building is considered to be located in the Guwahati city of India. The building plan and the extracted transverse frame considered for SeRA are shown in Figures 12 (a) and (b), respectively. The details of the assumed fibre discretization of beams and columns are depicted in Figures 12 (c) and (d), respectively. The NDSRA of the frame is executed using the OpenSees software

(McKenna et al. 2016). The concrete characteristic strength (f_{ck}), steel yield strength (f_y), and structural damping values (ξ) are considered to be random and assumed to be uncorrelated normal. Table 5 shows the statistical values of these parameters. The SeRA are performed for three structural performance levels i.e., the Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP). The permissible maximum storey drift ratio values for IO, LS and CP levels are 1%, 2% and 4%, respectively as per the FEMA-356 (2000).

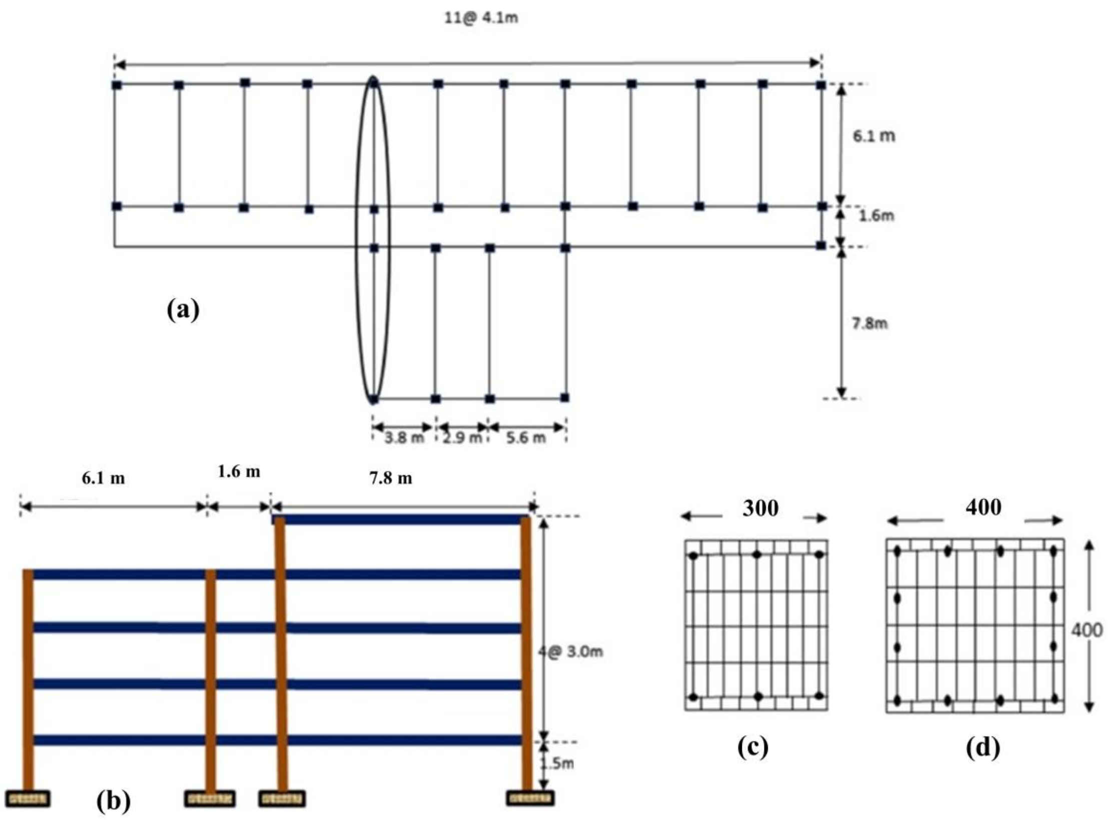

Figure 12 The details of the considered building: (a) The building plan, (b) the details of the extracted frame, the details of the fibre-discretization of (c) the beams and (d) the columns (Shyamal Ghosh, Ghosh, and Chakraborty 2018).

Table 5 The statistical properties of the random variables of the building frame

Random variables	Unit	Mean	Coefficient of variation	Truncation limits	
				Upper	Lower
f_{ck}	MPa	25	0.2	30	20
f_y	MPa	250	0.2	300	200
ξ	%	5	0.4	7	3

According to the UD table, $U_{30}(30^4)$, 30 training points are considered within the range of all four variables (f_{ck}, f_y, ξ and PGA) by arranging 30 equidistant levels of each variable to construct an initial DOE. Then, 24 responses at each of the 30 points are evaluated to construct 24 initial SVR models. Like the previous examples, P_f values are obtained by the proposed adaptive SVR approach, the active learning-based Kriging approach and the direct MCS technique considering 5000 samples at each PGA level. The number of maximum iterations required by the proposed adaptive SVR approach for ten different PGA values is shown in Figure 13. The observation is similar to the previous examples, i.e., these numbers do not follow any trend. Figure 14 shows the number of adaptive samples required by the proposed adaptive SVR approach for each case. For IO, LS and CP levels, the total number of adaptive samples required to obtain the final SVR model is 151, 110 and 99, respectively. Thus, 720 initial samples and 360 (= 151 + 110 + 99) adaptive samples, i.e., a total of 1080 (= 720 + 360) actual response evaluations are needed for the proposed adaptive SVR approach.

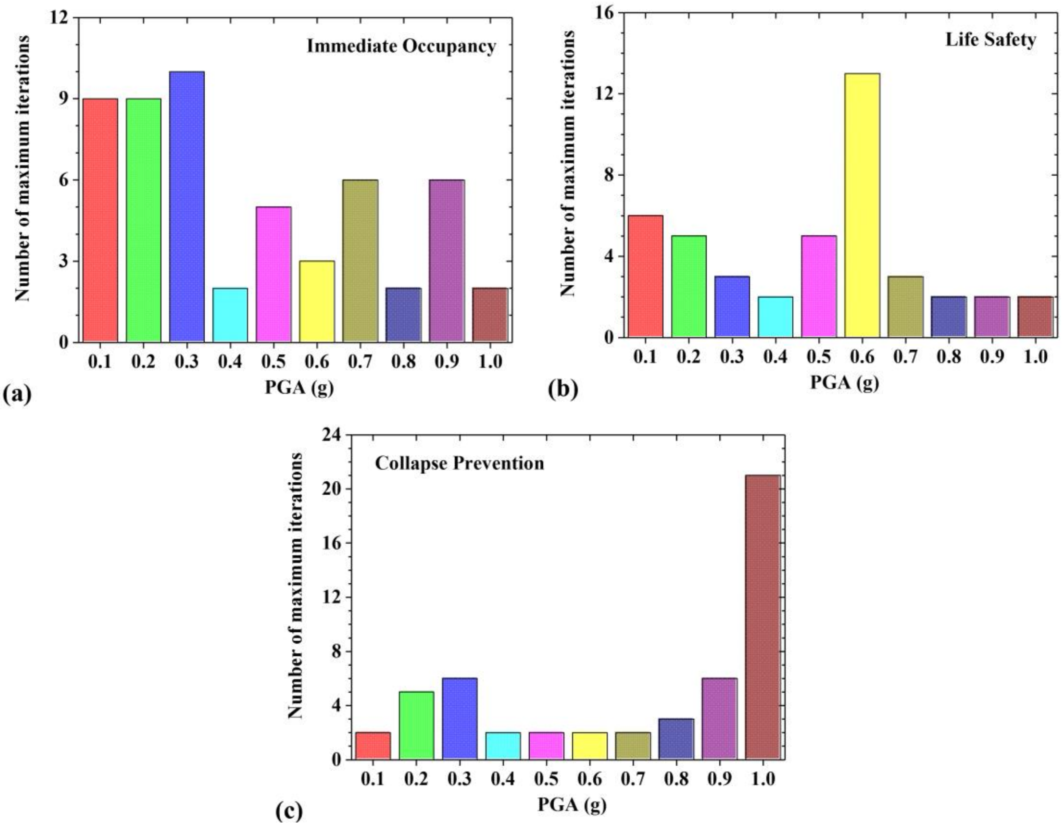

Figure 13 Numbers of maximum iterations required by the proposed adaptive SVR approach for SeRA of the building frame (a) at IO level, (b) at LS level and (c) at CP level.

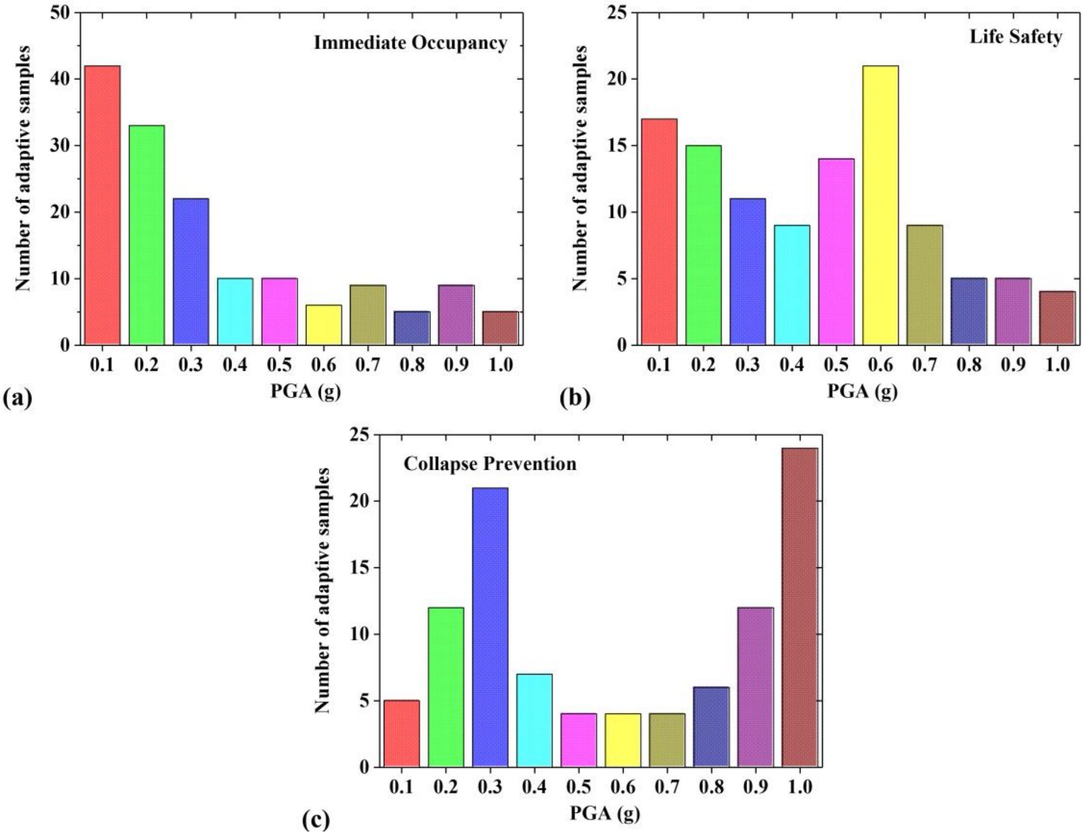

Figure 14 Numbers of adaptive samples added by the proposed adaptive SVR approach for SeRA of the building frame (a) at IO level, (b) at LS level and (c) at CP level.

The P_f values obtained by different approaches for three damage states with varying PGA levels are compared in Figure 15. The reliability results obtained by the proposed adaptive SVR approach are much closer to the reference results. On the other hand, the results of the active Kriging approach in many cases are far away from the direct MCS results. Like the previous examples, the computation time and the number of function evaluations required for each method are compared in Table 6. The direct MCS and the active Kriging approach require 50000 and 11424 function evaluations, respectively. Whereas only 1080 function evaluations are needed for the proposed adaptive SVR approach. Thus, the better efficiency and accuracy of the proposed adaptive SVR approach than the active learning-based Kriging approach is clearly observed for this example as well.

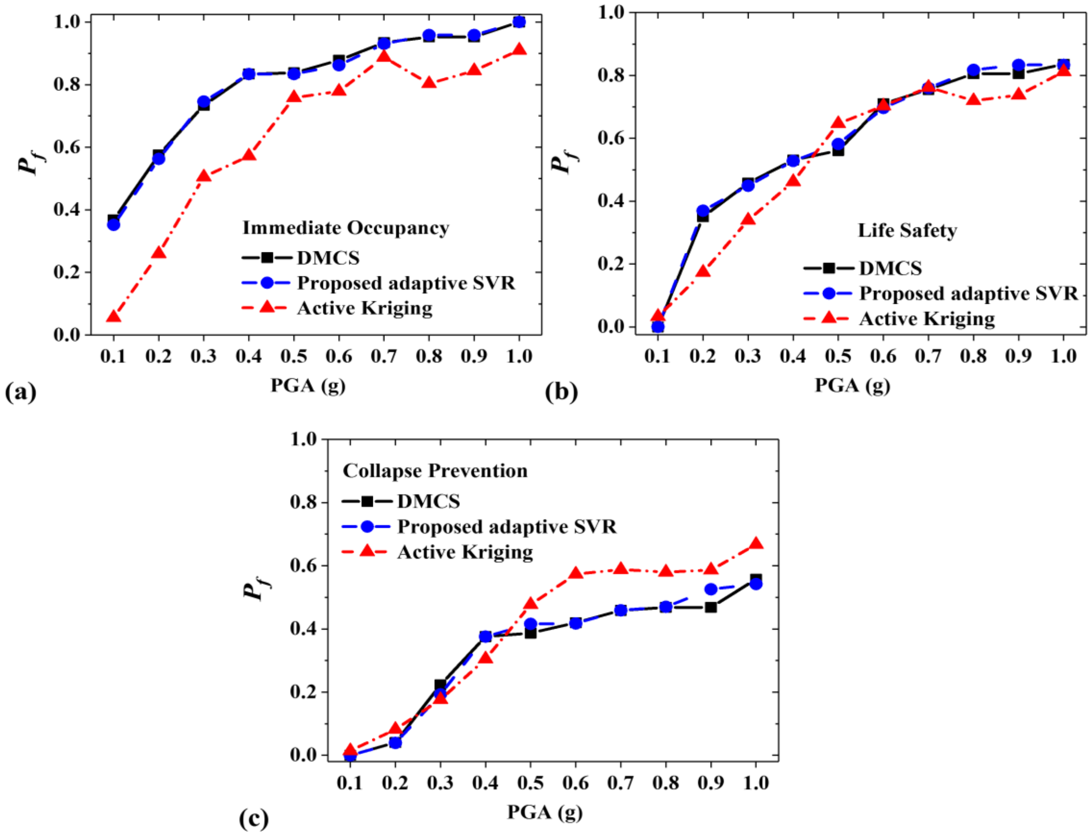

Figure 15 Comparisons of seismic reliability of the building frame (a) at IO level, (b) at LS level and (c) at CP level.

Table 6 Comparison of computation time and required number of actual function evaluations for SeRA of the building frame by different approaches

PGA	Proposed adaptive SVR		Active Kriging		DMCS	
	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations	Computation time (s)	No. of function evaluations
0.1 g	5944	784	7774	1848	33600	5000
0.2 g	5764	780	7742	1848	24098	5000
0.3 g	5761	774	7531	1800	31819	5000
0.4 g	5197	746	6695	1560	25704	5000
0.5 g	5177	748	7200	1704	19429	5000
0.6 g	5253	751	6826	1608	25889	5000
0.7 g	5140	742	8004	1920	33004	5000
0.8 g	4983	736	7849	1848	26909	5000
0.9 g	5194	746	8243	1920	34523	5000
1.0 g	5349	753	8183	1848	25440	5000
Total	19265	1080	41550	11424	280415	50000

6. Summary and conclusions

The present study proposed an adaptive metamodeling approach of SeRA where the metamodels are constructed directly to avoid the distribution assumption of seismic responses as is necessary in the usual dual metamodeling approach. The effectiveness of the proposed approach to estimate seismic reliability is demonstrated by considering the most accurate direct MCS-based results as the benchmark. In addition, results are also obtained by the state-of-the-art active learning-based Kriging approach for comparative study. The computation time and the number of function evaluations are noted for each example to compare the computational involvement of different approaches. The proposed adaptive SVR approach is observed to be quite efficient. The reliability results obtained by the proposed adaptive SVR approach are noted to be relatively close to the seismic reliability results obtained by the direct MCS technique in most of the PGA levels and damage states for all the examples studied. The improved performance of the proposed approach over the active learning-based Kriging approach in estimating seismic reliability is clearly observed. In fact, a large deviation is noted between the result obtained by the active learning-based Kriging approach and the direct MCS-based result in many cases. However, the approach is consistent and performs well for most of the PGA values at each damage state with a much smaller number of function evaluations. Thus, the approach seems to be an effective alternative for SeRA of structures. The proposed adaptive SVR approach is investigated for the SeRA of structures where the LSF is simplified by considering the maximum seismic response. However, it should be further explored for more general time-dependent SRA problems under other stochastic loadings e.g., wind, wave, blasts, etc. Active learning in combination with the proposed adaptive direct approach of response approximation can be explored further.

Data availability statement

The data that support the findings of this study are available from the corresponding author, SC, upon reasonable request.

Disclosure statement

The authors report that there are no competing interests to declare.

References

Afshari, Sajad Saraygord, Fatemeh Enayatollahi, Xiangyang Xu, and Xihui Liang. 2022. “Machine Learning-Based Methods in Structural Reliability Analysis: A Review.”

Reliability Engineering & System Safety 219 (March): 108223. https://doi.org/10.1016/J.RESS.2021.108223.

Balasubramanian, S. R., K. Balaji Rao, A. Meher Prasad, Rupen Goswami, and M. B. Anoop. 2014. "A Methodology for Development of Seismic Fragility Curves for URBM Buildings." *Earthquake and Structures* 6 (6): 611–25. https://doi.org/10.12989/eas.2014.6.6.611.

Bichon, B. J., M. S. Eldred, L. P. Swiler, S. Mahadevan, and J. M. McFarland. 2008. "Efficient Global Reliability Analysis for Nonlinear Implicit Performance Functions." *AIAA Journal* 46 (10): 2459–68. https://doi.org/10.2514/1.34321.

Bourinet, J. M. 2016. "Rare-Event Probability Estimation with Adaptive Support Vector Regression Surrogates." *Reliability Engineering and System Safety* 150 (June): 210–21. https://doi.org/10.1016/j.ress.2016.01.023.

Bucher, C.G., and U. Bourgund. 1990. "A Fast and Efficient Response Surface Approach for Structural Reliability Problems." *Structural Safety* 7 (1): 57–66. https://doi.org/10.1016/0167-4730(90)90012-E.

Buratti, N., B. Ferracuti, and M. Savoia. 2010. "Response Surface with Random Factors for Seismic Fragility of Reinforced Concrete Frames." *Structural Safety* 32 (1): 42–51. https://doi.org/10.1016/J.STRUSAFE.2009.06.003.

Cheng, Kai, and Zhenzhou Lu. 2020. "Structural Reliability Analysis Based on Ensemble Learning of Surrogate Models." *Structural Safety* 83 (March): 101905. https://doi.org/10.1016/j.strusafe.2019.101905.

Dai, Hongzhe, Boyi Zhang, and Wei Wang. 2015. "A Multiwavelet Support Vector Regression Method for Efficient Reliability Assessment." *Reliability Engineering and System Safety* 136 (April): 132–39. https://doi.org/10.1016/j.ress.2014.12.002.

Dai, Hongzhe, Hao Zhang, Wei Wang, and Guofeng Xue. 2012. "Structural Reliability Assessment by Local Approximation of Limit State Functions Using Adaptive Markov Chain Simulation and Support Vector Regression." *Computer-Aided Civil and Infrastructure Engineering* 27 (9): 676–86. https://doi.org/10.1111/j.1467-8667.2012.00767.x.

Echard, B., N. Gayton, and M. Lemaire. 2011. "AK-MCS: An Active Learning Reliability Method Combining Kriging and Monte Carlo Simulation." *Structural Safety* 33 (2):

145–54. https://doi.org/10.1016/j.strusafe.2011.01.002.

Fang, Kai-Tai, Dennis K.J. Lin, Peter Winker, and Yong Zhang. 2000. "Uniform Design: Theory and Application." *Technometrics* 42 (3): 237–48. https://doi.org/10.1080/00401706.2000.10486045.

FEMA-356. 2000. "Prestandard and Commentary for the Seismic Rehabilitation of Buildings." Washington, DC: Federal Emergency Management Agency (FEMA).

Franchin, P., A. Lupoi, P. E. Pinto, and M. Ij Schotanus. 2003. "Seismic Fragility of Reinforced Concrete Structures Using a Response Surface Approach." *Journal of Earthquake Engineering* 7 (SPEC. 1): 45–77. https://doi.org/10.1142/S1363246903000961.

Gaxiola-Camacho, J. Ramon, Hamoon Azizsoltani, Francisco J. Villegas-Mercado, and Achintya Haldar. 2017. "A Novel Reliability Technique for Implementation of Performance-Based Seismic Design of Structures." *Engineering Structures* 142 (July): 137–47. https://doi.org/10.1016/J.ENGSTRUCT.2017.03.076.

Gehl, P., and D. D'Ayala. 2016. "Development of Bayesian Networks for the Multi-Hazard Fragility Assessment of Bridge Systems." *Structural Safety* 60 (May): 37–46. https://doi.org/10.1016/J.STRUSAFE.2016.01.006.

Ghosh, Jayadipta, Jamie E. Padgett, and Leonardo Dueñas-Osorio. 2013. "Surrogate Modeling and Failure Surface Visualization for Efficient Seismic Vulnerability Assessment of Highway Bridges." *Probabilistic Engineering Mechanics* 34 (October): 189–99. https://doi.org/10.1016/J.PROBENGMECH.2013.09.003.

Ghosh, Shyamal, and Subrata Chakraborty. 2017. "Simulation Based Improved Seismic Fragility Analysis of Structures." *Earthquakes and Structures* 12 (5): 569–81. https://doi.org/10.12989/EAS.2017.12.5.569.

Ghosh, Shyamal, Swarup Ghosh, and Subrata Chakraborty. 2018. "Seismic Reliability Analysis of Reinforced Concrete Bridge Pier Using Efficient Response Surface Method–Based Simulation." *Advances in Structural Engineering* 21 (15): 2326–39. https://doi.org/10.1177/1369433218773422.

Ghosh, Shyamal, Atin Roy, and Subrata Chakraborty. 2018. "Support Vector Regression Based Metamodeling for Seismic Reliability Analysis of Structures." *Applied Mathematical Modelling* 64 (December): 584–602.

https://doi.org/10.1016/j.apm.2018.07.054.

———. 2021. “Kriging Metamodeling-Based Monte Carlo Simulation for Improved Seismic Fragility Analysis of Structures.” *Journal of Earthquake Engineering* 25 (7): 1316–36. https://doi.org/10.1080/13632469.2019.1570395.

Ghosh, Swarup. 2020. “Seismic Fragility Analysis of Structures with Special Emphasis to Northeast India.” PhD thesis, Indian Institute of Engineering Science and Technology, Shibpur.

Gidaris, Ioannis, Alexandros A. Taflanidis, and George P. Mavroeidis. 2015. “Kriging Metamodeling in Seismic Risk Assessment Based on Stochastic Ground Motion Models.” *Earthquake Engineering & Structural Dynamics* 44 (14): 2377–99. https://doi.org/10.1002/EQE.2586.

Günay, Selim, and Khalid M. Mosalam. 2013. “PEER Performance-Based Earthquake Engineering Methodology, Revisited.” *Journal of Earthquake Engineering* 17 (6): 829–58. https://doi.org/10.1080/13632469.2013.787377.

Hurtado, Jorge E. 2004. “An Examination of Methods for Approximating Implicit Limit State Functions from the Viewpoint of Statistical Learning Theory.” *Structural Safety* 26 (3): 271–93. https://www.sciencedirect.com/science/article/pii/S0167473003000420.

———. 2007. “Filtered Importance Sampling with Support Vector Margin: A Powerful Method for Structural Reliability Analysis.” *Structural Safety* 29 (1): 2–15. https://linkinghub.elsevier.com/retrieve/pii/S0167473005000597.

Jafari-Asl, Jafar, Sima Ohadi, Mohamed El Amine Ben Seghier, and Nguyen-Thoi Trung. 2021. “Accurate Structural Reliability Analysis Using an Improved Line-Sampling-Method-Based Slime Mold Algorithm.” *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering* 7 (2): 04021015. https://doi.org/10.1061/AJRUA6.0001129.

Jafari-Asl, Jafar, Mohamed El Amine Ben Seghier, Sima Ohadi, José Correia, and João Barroso. 2022. “Reliability Analysis Based Improved Directional Simulation Using Harris Hawks Optimization Algorithm for Engineering Systems.” *Engineering Failure Analysis* 135 (May): 106148. https://doi.org/10.1016/J.ENGFAILANAL.2022.106148.

Jeddi, Ashkan B., Abdollah Shafieezadeh, Jieun Hur, Jeong Gon Ha, Daegi Hahm, and Min Kyu Kim. 2022. “Multi-Hazard Typhoon and Earthquake Collapse Fragility Models for

Transmission Towers: An Active Learning Reliability Approach Using Gradient Boosting Classifiers.” *Earthquake Engineering & Structural Dynamics* 51 (15): 3552–73. https://doi.org/10.1002/EQE.3735.

Johnson, M.E., L.M. Moore, and D. Ylvisaker. 1990. “Minimax and Maximin Distance Designs.” *Journal of Statistical Planning and Inference* 26 (2): 131–48. https://doi.org/10.1016/0378-3758(90)90122-B.

Jones, Donald R, Matthias Schonlau, and W. J. Welch. 1998. “Efficient Global Optimization of Expensive Black-Box Functions," , Vol. 13, No. 4, Pp. 455-492, 1998.” *Journal of Global Optimization* 13: 455–92. https://link.springer.com/content/pdf/10.1023%2FA%3A1008306431147.pdf.

Kazantzi, A. K., T. D. Righiniotis, and M. K. Chryssanthopoulos. 2008. “Fragility and Hazard Analysis of a Welded Steel Moment Resisting Frame.” *Journal of Earthquake Engineering* 12 (4): 596–615. https://doi.org/10.1080/13632460701512993.

Keshtegar, Behrooz, Mohamed El Amine Ben Seghier, Enrico Zio, José A.F.O. Correia, Shun Peng Zhu, and Nguyen Thoi Trung. 2021. “Novel Efficient Method for Structural Reliability Analysis Using Hybrid Nonlinear Conjugate Map-Based Support Vector Regression.” *Computer Methods in Applied Mechanics and Engineering* 381 (August): 113818. https://doi.org/10.1016/J.CMA.2021.113818.

Kim, Sang Hoon, and Maria Q. Feng. 2003. “Fragility Analysis of Bridges under Ground Motion with Spatial Variation.” *International Journal of Non-Linear Mechanics* 38 (5): 705–21. https://doi.org/10.1016/S0020-7462(01)00128-7.

Kundu, Anirban, Swarup Ghosh, and Subrata Chakraborty. 2022. “A Long Short-Term Memory Based Deep Learning Algorithm for Seismic Response Uncertainty Quantification.” *Probabilistic Engineering Mechanics* 67 (January): 103189. https://doi.org/10.1016/j.probengmech.2021.103189.

Kwon, Oh Sung, and Amr Elnashai. 2006. “The Effect of Material and Ground Motion Uncertainty on the Seismic Vulnerability Curves of RC Structure.” *Engineering Structures* 28 (2): 289–303. https://doi.org/10.1016/J.ENGSTRUCT.2005.07.010.

Lagaros, Nikolaos D., and Michalis Fragiadakis. 2007. “Fragility Assessment of Steel Frames Using Neural Networks.” *Earthquake Spectra* 23 (4): 735–52. https://doi.org/10.1193/1.2798241.

Lagaros, Nikos D., Yiannis Tsompanakis, Prodromos N. Psarropoulos, and Evaggelos C. Georgopoulos. 2009. "Computationally Efficient Seismic Fragility Analysis of Geostructures." *Computers and Structures* 87 (19–20): 1195–1203. https://doi.org/10.1016/j.compstruc.2008.12.001.

Li, Hong-shuang Shuang, Zhen-zhou Lü, Zhu-feng Feng Yue, Zhen Zhou Lu, and Zhu-feng Feng Yue. 2006. "Support Vector Machine for Structural Reliability Analysis." *Applied Mathematics and Mechanics (English Edition)* 27 (10): 1295–1303. https://doi.org/10.1007/s10483-006-1001-z.

Li, Xu, Chunlin Gong, Liangxian Gu, Wenkun Gao, Zhao Jing, and Hua Su. 2018. "A Sequential Surrogate Method for Reliability Analysis Based on Radial Basis Function." *Structural Safety* 73 (July): 42–53. https://doi.org/10.1016/j.strusafe.2018.02.005.

Lin, Dennis K.J., and Wanzhu Tu. 1995. "Dual Response Surface Optimization." *Journal of Quality Technology* 27 (1): 34–39. https://doi.org/10.1080/00224065.1995.11979556.

Mai, Chu V., Minas D. Spiridonakos, Eleni N. Chatzi, and Bruno Sudret. 2016. "Surrogate Modelling for Stochastic Dynamical Systems by Combining Narx Models and Polynomial Chaos Expansions." *International Journal for Uncertainty Quantification* 6 (4): 313–39. https://doi.org/10.1615/INT.J.UNCERTAINTYQUANTIFICATION.2016016603.

Mangalathu, Sujith, Jong Su Jeon, and Reginald DesRoches. 2018. "Critical Uncertainty Parameters Influencing Seismic Performance of Bridges Using Lasso Regression." *Earthquake Engineering & Structural Dynamics* 47 (3): 784–801. https://doi.org/10.1002/EQE.2991.

Marelli, Stefano, and Bruno Sudret. 2018. "An Active-Learning Algorithm That Combines Sparse Polynomial Chaos Expansions and Bootstrap for Structural Reliability Analysis." *Structural Safety* 75 (November): 67–74. https://doi.org/10.1016/j.strusafe.2018.06.003.

McKenna, F., GL. Fenves, MH. Scott, and B. Jeremic. 2016. "OpenSees: Open System for Earthquake Engineering Simulation. Http://Opensees.Berkeley.Edu." Berkeley, CA: Pacific Earthquake Engineering Research Center, University of California.

Möller, Oscar, Ricardo O. Foschi, Marcelo Rubinstein, and Laura Quiroz. 2009. "Seismic Structural Reliability Using Different Nonlinear Dynamic Response Surface Approximations." *Structural Safety* 31 (5): 432–42. https://doi.org/10.1016/J.STRUSAFE.2008.12.001.

Moura, Márcio das Chagas, Enrico Zio, Isis Didier Lins, and Enrique Droguett. 2011. "Failure and Reliability Prediction by Support Vector Machines Regression of Time Series Data." *Reliability Engineering & System Safety* 96 (11): 1527–34. https://doi.org/10.1016/J.RESS.2011.06.006.

Moustapha, Maliki, Stefano Marelli, and Bruno Sudret. 2022. "Active Learning for Structural Reliability: Survey, General Framework and Benchmark." *Structural Safety* 96 (May): 102174. https://doi.org/10.1016/J.STRUSAFE.2021.102174.

O'Reilly, Gerard J., and Timothy J. Sullivan. 2016. "Fragility Functions for Eccentrically Braced Steel Frame Structures." *Earthquake and Structures* 10 (2): 367–88. https://doi.org/10.12989/eas.2016.10.2.367.

Pan, Qiujing, and Daniel Dias. 2017. "An Efficient Reliability Method Combining Adaptive Support Vector Machine and Monte Carlo Simulation." *Structural Safety* 67 (July): 85–95. https://doi.org/10.1016/j.strusafe.2017.04.006.

Park, Joonam, and Peeranan Towashiraporn. 2014. "Rapid Seismic Damage Assessment of Railway Bridges Using the Response-Surface Statistical Model." *Structural Safety* 47 (March): 1–12. https://doi.org/10.1016/J.STRUSAFE.2013.10.001.

Pinto, Paolo Emilio. 2001. "Reliability Methods in Earthquake Engineering." *Progress in Structural Engineering and Materials* 3 (1): 76–85. https://doi.org/10.1002/PSE.64.

Porter, Keith, Robert Kennedy, and Robert Bachman. 2007. "Creating Fragility Functions for Performance-Based Earthquake Engineering." *Earthquake Spectra* 23 (2): 471–89. https://doi.org/10.1193/1.2720892.

Rajashekhar, Malur R., and Bruce R. Ellingwood. 1993. "A New Look at the Response Surface Approach for Reliability Analysis." *Structural Safety* 12 (3): 205–20. https://doi.org/10.1016/0167-4730(93)90003-J.

Ren, Chao, Younes Aoues, Didier Lemosse, and Eduardo Souza De Cursi. 2022. "Ensemble of Surrogates Combining Kriging and Artificial Neural Networks for Reliability Analysis with Local Goodness Measurement." *Structural Safety* 96 (January): 102186. https://doi.org/10.1016/j.strusafe.2022.102186.

Richard, Benjamin, Christian Cremona, and Lucas Adelaide. 2012. "A Response Surface Method Based on Support Vector Machines Trained with an Adaptive Experimental Design." *Structural Safety* 39 (November): 14–21.

https://doi.org/10.1016/j.strusafe.2012.05.001.

Rocco, Claudio M., and José Alí Moreno. 2002. "Fast Monte Carlo Reliability Evaluation Using Support Vector Machine." *Reliability Engineering and System Safety* 76 (3): 237–43. https://doi.org/10.1016/S0951-8320(02)00015-7.

Roy, Atin, and Subrata Chakraborty. 2020. "Support Vector Regression Based Metamodel by Sequential Adaptive Sampling for Reliability Analysis of Structures." *Reliability Engineering and System Safety* 200 (August): 106948. https://doi.org/10.1016/j.ress.2020.106948.

———. 2022. "Reliability Analysis of Structures by a Three-Stage Sequential Sampling Based Adaptive Support Vector Regression Model." *Reliability Engineering and System Safety* 219 (March): 108260. https://doi.org/10.1016/j.ress.2021.108260.

———. 2023. "Support Vector Machine in Structural Reliability Analysis: A Review." *Reliability Engineering & System Safety* 233 (May): 109126. https://doi.org/10.1016/j.ress.2023.109126.

Roy, Atin, Subrata Chakraborty, and Sondipon Adhikari. 2023. "Reliability Analysis of Structures by Active Learning Enhanced Sparse Bayesian Regression." *Journal of Engineering Mechanics* 149 (5): 04023024. https://doi.org/10.1061/jenmdt.emeng-6964.

Roy, Atin, Ramkrishna Manna, and Subrata Chakraborty. 2019. "Support Vector Regression Based Metamodeling for Structural Reliability Analysis." *Probabilistic Engineering Mechanics* 55 (January): 78–89. https://doi.org/10.1016/j.probengmech.2018.11.001.

Saha, Sandip Kumar, Vasant Matsagar, and Subrata Chakraborty. 2016. "Uncertainty Quantification and Seismic Fragility of Base-Isolated Liquid Storage Tanks Using Response Surface Models." *Probabilistic Engineering Mechanics* 43 (January): 20–35. https://doi.org/10.1016/J.PROBENGMECH.2015.10.008.

Sainct, Rémi, Cyril Feau, Jean Marc Martinez, and Josselin Garnier. 2020. "Efficient Methodology for Seismic Fragility Curves Estimation by Active Learning on Support Vector Machines." *Structural Safety* 86 (September): 101972. https://doi.org/10.1016/j.strusafe.2020.101972.

Santner, Thomas J., Brian J. Williams, and William I. Notz. 2003. "Space-Filling Designs for Computer Experiments." In , 121–61. Springer, New York, NY. https://doi.org/10.1007/978-1-4757-3799-8_5.

Schölkopf, Bernhard., Christopher J. C. Burges, and Alexander J. Smola. 1999. *Advances in Kernel Methods : Support Vector Learning*. MIT Press. https://dl.acm.org/citation.cfm?id=299094.

Segura, Rocio, Jamie E. Padgett, and Patrick Paultre. 2020. "Metamodel-Based Seismic Fragility Analysis of Concrete Gravity Dams." *Journal of Structural Engineering* 146 (7): 04020121. https://doi.org/10.1061/(ASCE)ST.1943-541X.0002629.

Seo, Junwon, Leonardo Dueñas-Osorio, James I. Craig, and Barry J. Goodno. 2012. "Metamodel-Based Regional Vulnerability Estimate of Irregular Steel Moment-Frame Structures Subjected to Earthquake Events." *Engineering Structures* 45 (December): 585–97. https://doi.org/10.1016/J.ENGSTRUCT.2012.07.003.

Seo, Junwon, and Daniel G. Linzell. 2013. "Use of Response Surface Metamodels to Generate System Level Fragilities for Existing Curved Steel Bridges." *Engineering Structures* 52 (July): 642–53. https://doi.org/10.1016/J.ENGSTRUCT.2013.03.023.

Smola, Alex J., and Bernhard Schölkopf. 2004. "A Tutorial on Support Vector Regression." *Statistics and Computing* 14 (3): 199–222. https://doi.org/10.1023/B:STCO.0000035301.49549.88.

Teixeira, Rui, Maria Nogal, and Alan O'Connor. 2021. "Adaptive Approaches in Metamodel-Based Reliability Analysis: A Review." *Structural Safety* 89 (March): 102019. https://doi.org/10.1016/j.strusafe.2020.102019.

Towashiraporn, Peeranan. 2004. "Building Seismic Fragilities Using Response Surface Metamodels." *Georgia Institute of Technology*. PhD thesis, Georgia Institute of Technology. https://search.proquest.com/openview/c9295fb2060f2854f76a5457a5db79b0/1?pq-origsite=gscholar&cbl=18750&diss=y.

Unnikrishnan, V. U., A. M. Prasad, and B. N. Rao. 2013. "Development of Fragility Curves Using High-Dimensional Model Representation." *Earthquake Engineering & Structural Dynamics* 42 (3): 419–30. https://doi.org/10.1002/EQE.2214.

Vapnik, Vladimir N. 1995. *The Nature of Statistical Learning Theory*. Springer. https://dl.acm.org/citation.cfm?id=211359.

———. 1998. *Statistical Learning Theory*. Wiley.

———. 2000. *The Nature of Statistical Learning Theory*. Springer.

Wang, Zhiyi, Nicola Pedroni, Irmela Zentner, and Enrico Zio. 2018. "Seismic Fragility Analysis with Artificial Neural Networks: Application to Nuclear Power Plant Equipment." *Engineering Structures* 162 (May): 213–25. https://doi.org/10.1016/J.ENGSTRUCT.2018.02.024.

Wong, S. M., R. E. Hobbs, and C. Onof. 2005. "An Adaptive Response Surface Method for Reliability Analysis of Structures with Multiple Loading Sequences." *Structural Safety* 27 (4): 287–308. https://doi.org/10.1016/j.strusafe.2005.02.001.

Xiao, Ning Cong, Ming J. Zuo, and Chengning Zhou. 2018. "A New Adaptive Sequential Sampling Method to Construct Surrogate Models for Efficient Reliability Analysis." *Reliability Engineering and System Safety* 169 (January): 330–38. https://doi.org/10.1016/j.ress.2017.09.008.

Xiao, Yan, Kun Ye, and Wenfu He. 2020. "An Improved Response Surface Method for Fragility Analysis of Base-Isolated Structures Considering the Correlation of Seismic Demands on Structural Components." *Bulletin of Earthquake Engineering* 18 (8): 4039–59. https://doi.org/10.1007/s10518-020-00836-w.

Xiao, Yanjie, Feng Yue, and Xun'An Zhang. 2021. "Seismic Fragility Analysis of Structures Based on Adaptive Gaussian Process Regression Metamodel." *Shock and Vibration* 2021. https://doi.org/10.1155/2021/7622130.

Xiao, Yanjie, Xun'An Zhang, Feng Yue, Muhammad Moman Shahzad, Xinwei Wang, and Buqiao Fan. 2022. "Seismic Fragility Analysis of Mega-Frame with Vibration Control Substructure Based on Dual Surrogate Model and Active Learning." *Buildings 2022, Vol. 12, Page 752* 12 (6): 752. https://doi.org/10.3390/BUILDINGS12060752.

Xu, Zhaoyi, and Joseph Homer Saleh. 2021. "Machine Learning for Reliability Engineering and Safety Applications: Review of Current Status and Future Opportunities." *Reliability Engineering and System Safety* 211 (August 2020): 107530. https://doi.org/10.1016/j.ress.2021.107530.

Zentner, Irmela, and Emanuele Borgonovo. 2014. "Construction of Variance-Based Metamodels for Probabilistic Seismic Analysis and Fragility Assessment." *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards* 8 (3): 202–16. https://doi.org/10.1080/17499518.2014.958173.

Zhang, Yuye, and Gang Wu. 2019. "Seismic Vulnerability Analysis of RC Bridges Based on Kriging Model." *Journal of Earthquake Engineering* 23 (2): 242–60.

https://doi.org/10.1080/13632469.2017.1323040.

Zhou, Tong, Yongbo Peng, and Jie Li. 2019. “An Efficient Reliability Method Combining Adaptive Global Metamodel and Probability Density Evolution Method.” *Mechanical Systems and Signal Processing* 131 (September): 592–616. https://doi.org/10.1016/J.YMSSP.2019.06.009.

Zhu, Shun Peng, Behrooz Keshtegar, Mohamed El Amine Ben Seghier, Enrico Zio, and Osman Taylan. 2022. “Hybrid and Enhanced PSO: Novel First Order Reliability Method-Based Hybrid Intelligent Approaches.” *Computer Methods in Applied Mechanics and Engineering* 393 (April): 114730. https://doi.org/10.1016/J.CMA.2022.114730.

Appendix

A. Support vector regression

For *p* number of training data pairs, $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_p, y_p)\}$, $\mathbf{x} \subset \mathbf{R}^n$ and $y \subset \mathbf{R}$, where, **x** is the input vector, *y* is the corresponding output, **R** denotes the set of all real numbers; *n* is the input dimension. For a linear mapping, the regression function f is expressed as,

$$f(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b, \qquad \mathbf{w} \in \mathbf{R}^n, \qquad b \in \mathbf{R} \tag{A.1}$$

where, $\langle \mathbf{w}, \mathbf{x} \rangle$ is the dot product of $\mathbf{w}$ and $\mathbf{x}$; $\mathbf{w}$ and b represent weight vector and bias, respectively. The best f is mathematically searched by minimizing the norm $\|\mathbf{w}\|^2 = \langle \mathbf{w}, \mathbf{w} \rangle$ and, for this, the optimization problem is defined as follows,

$$\min \frac{1}{2}\|\mathbf{w}\|^2 \qquad \text{s.t.} \quad \begin{cases} y_i - \langle \mathbf{w}, \mathbf{x}_i \rangle - b \le \varepsilon \\ \langle \mathbf{w}, \mathbf{x}_i \rangle + b - y_i \le \varepsilon \end{cases} \tag{A.2}$$

where, ε is a non-negative precision tolerance. However, this optimization problem is to be only applicable if the function f can approximate the output at all the training samples within $\pm\varepsilon$ from the actual value. Nevertheless, the above condition can be hardly achievable for all the training points. Therefore, two slack variables ξ_i, ξ_i^* are introduced in the above optimization problem to relax the precision tolerance. The modified optimization problem is expressed as follows (Vapnik 1995),

$$\min \ \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{p}\left(\xi_i + \xi_i^*\right) \quad \text{s.t.} \quad \begin{cases} y_i - \langle \mathbf{w}, \mathbf{x}_i \rangle - b \le \varepsilon + \xi_i, \\ \langle \mathbf{w}, \mathbf{x}_i \rangle + b - y_i \le \varepsilon + \xi_i^*, \end{cases} \quad \xi_i, \xi_i^* \ge 0 \tag{A.3}$$

where, C is the regularization constant that regulates the trade-off between the flatness of f (represented by the norm $\|\mathbf{w}\|^2$) and the relaxation on the fitting error.

For brevity, the final outcome of Eq. (A.3) is directly provided here. The detailed solution procedure may be seen in Smola and Schölkopf (2004). The best regression function $f(\mathbf{x})$ is obtained as follows,

$$f(\mathbf{x}) = \sum_{i=1}^{p}\left(\alpha_i - \alpha_i^*\right)\langle \mathbf{x}_i, \mathbf{x} \rangle + b, \qquad \alpha_i, \alpha_i^* \in [0, C] \tag{A.4}$$

where, α_i and α_i^* are the Lagrange dual variables (Smola and Schölkopf 2004). The SVR method can be readily extended to nonlinear regression cases by replacing the dot product $\langle \mathbf{x}, \mathbf{x}_i \rangle$ in Eq. (A.4) with a kernel function $K(\mathbf{x}, \mathbf{x}_i)$ as,

$$f(\mathbf{x}) = \sum_{i=1}^{p}\left(\alpha_i - \alpha_i^*\right)K(\mathbf{x}_i, \mathbf{x}) + b. \tag{A.5}$$

A function which satisfies the Mercer's condition can be selected as the kernel function (Schölkopf, Burges, and Smola 1999). The GRBF kernel adopted in the present study is defined as follows,

$$K\left(\mathbf{x},\mathbf{x}_i\right)=\exp\left(-\frac{\|\mathbf{x}-\mathbf{x}_i\|^2}{2\sigma^2}\right) \tag{A.6}$$

where, σ is a parameter of the GRBF kernel function.